ICRM Gamma Spectrometry Working Group

Workshop

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Detection limit – Decision threshold Application of ISO 11929

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ISO standards on determination of the Detection Limit and Decision threshold for ionising radiation measurements

ISO/CD 11929-1: Determination of the Detection Limit and Decision Threshold for Ionizing Radiation Measurements - Part 1: Fundamentals and Applications to Counting Measurements without the Influence of Sample Treatment.

ISO/CD 11929-2: Determination of the Detection Limit and Decision Threshold for Ionizing Radiation Measurements - Part 2: Fundamentals and Applications to Counting Measurements with the Influence of Sample Treatment.

ISO/CD 11929-3: Determination of the Detection Limit and Decision Threshold for Ionizing Radiation Measurements - Part 3: Fundamentals and Applications to Counting Measurements by High Resolution Gamma Spectrometry, without the Influence of Sample Treatment.

ISO/CD 11929-4: Determination of the Detection Limit and Decision Threshold for Ionizing Radiation Measurements - Part 4: Fundamentals and Applications to Measurements by Use of Linear Analogue Ratemeters, without the Influence of Sample Treatment.

ISO/CD 11929-5: Determination of the Detection Limit and Decision Threshold for Ionizing Radiation Measurements - Part 5: Fundamentals and Applications to Measurements of Filters During Accumulation of Radioactive Materials.

ISO/CD 11929-6: Determination of the Detection Limit and Decision Threshold for Ionizing Radiation Measurements - Part 6: Fundamentals and Applications to Measurements by Use of a Transient Measuring Mode.

ISO/CD 11929-7: Determination of the Detection Limit and Decision Threshold for Ionizing Radiation Measurements - Part 7: Fundamentals and General Applications.

ISO/CD 11929-8: Determination of the Detection Limit and Decision Threshold for Ionizing Radiation Measurements - Part 8: Fundamentals and Application to Unfolding of Spectrometric Measurements without the Influence of Sample Treatment.

Detection Limit and Decision threshold: Definitions

DECISION QUANTITY: random variable for the decision whether the physical effect to be measured is present or not

DECISION THRESHOLD: fixed value of the decision quantity by which, when exceeded by the result of an actual measurement of a measurand quantifying a physical effect, one decides that the physical effect is present

DETECTION LIMIT: smallest value of the measurand which is detectable by the measuring method

CONFIDENCE INTERVAL: values which define confidence intervals to be specified for the measurand in question which, if the result exceeds the decision threshold, includes the true value of the measurand with a given probability

Three simple questions in particle counting

N_0 : BACKGROUND counts

N_s : SAMPLE counts

QUESTION N°1: Which is the value of the net count that, when exceeded by the result of an actual measurement, one decides that there is a real contribution from the sample ?

QUESTION N°2: Which is the smallest value of the sample contribution that is detectable by the measuring system ?

QUESTION N° 3: If such a contribution has been detected, which is the interval that includes the true value with a given probability ?

Possible answer to question N. 1

- Compare N_s with N_0, considering the statistical fluctuation of N_0:

  - o $Var(N_0) = N_0$ (Poisson)
  - o $L = 3\,(N_0)^{1/2}$ (3σ Criteria, ICRU 22, 1979)

- PROBLEMS:

  1: arbitrary choice of the factor 3;

  2: NS>L: can be a background fluctuation ?

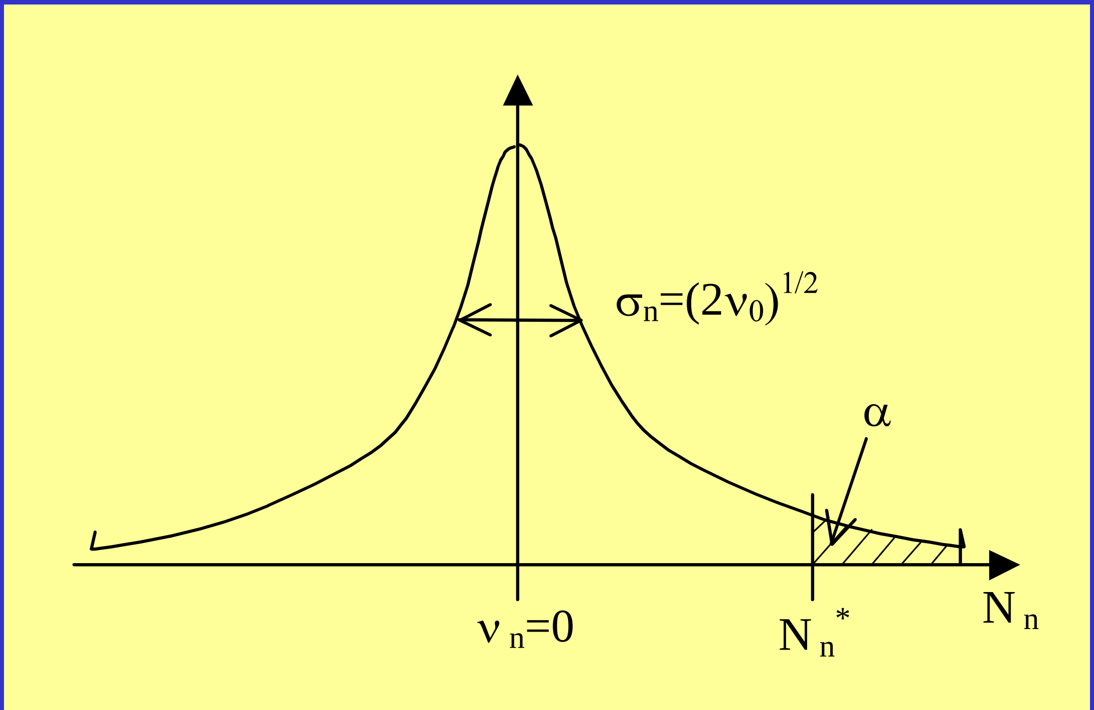
  3: NS<N0: there is no contribution from the sample ?

- A MORE RIGOROUS ANSWER IS NEEDED: →

Answer to question N. 1 (Decision threshold)

	COUNTS	EXPECTATION VALUE	STANDARD DEVIATION
BACKGROUND	N_0	υ_0	$\sigma_0 = \sqrt{\upsilon_0}$
SAMPLE	N_s	υ_s	$\sigma_s = \sqrt{\upsilon_s}$
NET	$N_n = N_s - N_0$	$\upsilon_n = \upsilon_s - \upsilon_0$	$\sigma_n = \sqrt{\upsilon_s + \upsilon_0}$

HP: $\nu_n=0$ ($\nu_s=\nu_0$) $\sigma_n=(2\nu_0)^{1/2}$ NO contribution from sample



DECISION THRESHOLD:

$N_n^*=K_\alpha\sigma_n$ (Currie, 1968)

THE DECISION THRESHOLD is the critical value for the statistical test for the decision between the hypothesis that the sample effect is not present and the alternative hypothesis that it is present. When the critical value is exceeded by the result of an actual measurement this is taken to indicate that the hypothesis should be rejected. The statistical test shall be designed such that the probability of wrongly rejecting the hypothesis (error of the first kind) is equal to a given value α.

α	Kα
0,1	1,28
0,05	1,64
0,025	1,96
0,001	3,09

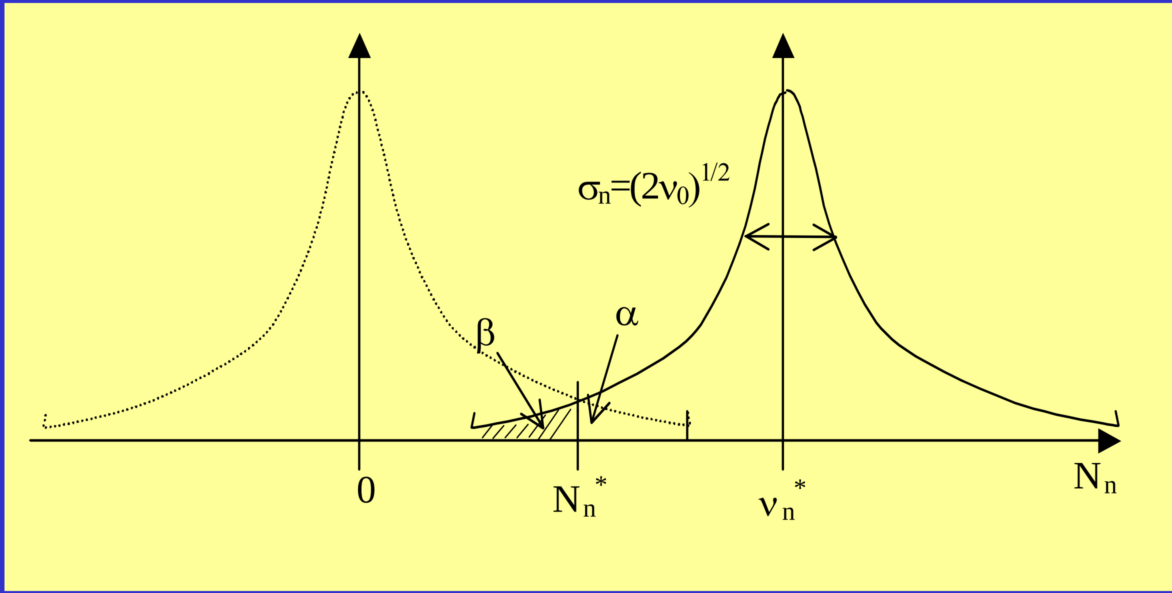
$N_n^* = k_\alpha \sigma_n$ $\sigma_n = \sqrt{2\upsilon_0}$

$\alpha = 0.05 \Rightarrow N_n^* = 2.32\sqrt{\upsilon_0}$

Answer to question N. 2 (Detection limit)

	COUNTS	EXPECTATION VALUE	STANDARD DEVIATION
BACKGROUND	N_0	υ_0	$\sigma_0 = \sqrt{\upsilon_0}$
SAMPLE	N_s	υ_s	$\sigma_s = \sqrt{\upsilon_s}$
NET	$N_n = N_s - N_0$	$\upsilon_n = \upsilon_s - \upsilon_0$	$\sigma_n = \sqrt{\upsilon_s + \upsilon_0}$

HP: $\nu_n \neq 0$ $(\nu_s > \nu_0)$ $\sigma_n = (2\nu_0)^{1/2}$ **YES contribution from sample**



DECISION THRESHOLD:

$\nu_n^* = N_n + K_\beta \sigma_n = (K_\alpha + K_\beta)\,\sigma_n$ (Currie, 1968)

THE DETECTION LIMIT is the smallest true value of the measurand which is associated with the statistical test and hypothesis (made for the decision threshold) by the following characteristics: If in reality the true value is equal to or exceeds the detection limit, the probability of wrongly not rejecting the hypothesis (error of the second kind) shall be at most equal to a given value b.

α	Kα
0,1	1,28
0,05	1,64
0,025	1,96
0,001	3,09

$$\sigma_n = \sqrt{2\upsilon_0}$$

$$\left.\begin{array}{l}\alpha = 0.05\\ \beta = 0.05\end{array}\right\} \Rightarrow \upsilon_n^* = 4.64\sqrt{\upsilon_0}$$

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Generalization (a): Test of Hypothesis

HYPOTHESIS H_0: No sample contribution to the count.

	H_0 accepted	H_0 rejected
Ho true	OK $P=1-\alpha$	Type I error $P=\alpha$
Ho false	Type II error $P=\beta$	OK $P=1-\beta$

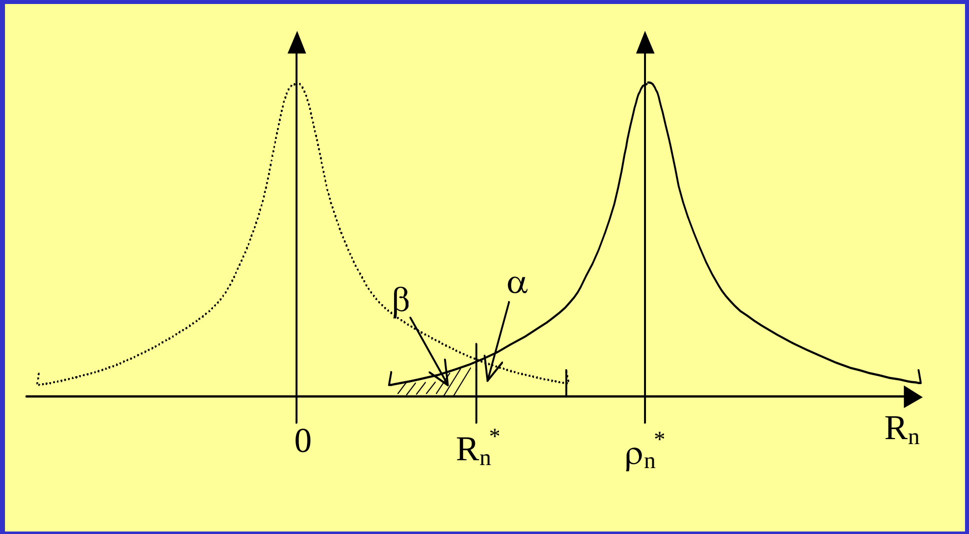
α: Probability of rejecting the hypothesis H_0 when, in realty, it is true

β: Probability of accepting the hypothesis H_0 when, in realty, it is false

Analogue considerations can be made in case of counting with preset count condition

Generalization (b): Definitions

	COUNTS	COUNTING TIME	COUNT RATE	EXPECTATION VALUE	STANDARD DEVIATION
BACKGROUND	N_0	t_0	R_0	ρ_0	σ_0
SAMPLE	N_s	t_s	R_s	ρ_s	σ_s
NET	-	-	$R_n = R_s - R_0$	$\rho_n = \rho_s - \rho_0$	σ_n



DECISION THRESHOLD: Critical value R_n^* of the statistical test for the decision between the alternative hypothesis:

A) $\rho_s=\rho_0$

B) $\rho_s>\rho_0$

with given probability α of type I error:

$$R_n^*=k_\alpha \sigma_n.$$

DETECTION LIMIT: smallest expectation value ρ_0^*, associated to the statistical test between the hypothesis A and B above, which determines a type II error with given probability β.

$$\rho_n^*=(k_\alpha+ k_\beta)\sigma_n$$

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Generalization (c): Use of R_n^* and ρ_n^*

- R_n^* should be compared with measurement results to assess weather a sample contribution has been detected (a-posteriori criteria):

  $R_n > R_n^*$ sample contribution detected;

  $R_n < R_n^*$ sample contribution not detected.

- ρ_0^* should be used to check weather a measuring procedure is suitable for the purpose of the measurement. It should be compared with a specific guideline value S(β) as specific requirements on the sensitivity of the measuring procedure for scientific, legal or other reasons (a-priori criteria):

  $\rho_0 > \rho_0^*$ the sample contribution will be detected with probability greater than 1-β;

  $\rho_0 < \rho_0^*$ the sample contribution will be detected with probability less than 1-β.

  $\rho_0 < S(b)$ measurement procedure is not adequate for the intended purpose.

- When reporting decision threshold and detection limits it is important to give the values of α and β used.

- "All knowledge is divided into two categories: a priori and a posteriori knowledge", I. Kant, Critique of Pure Reason (1791).

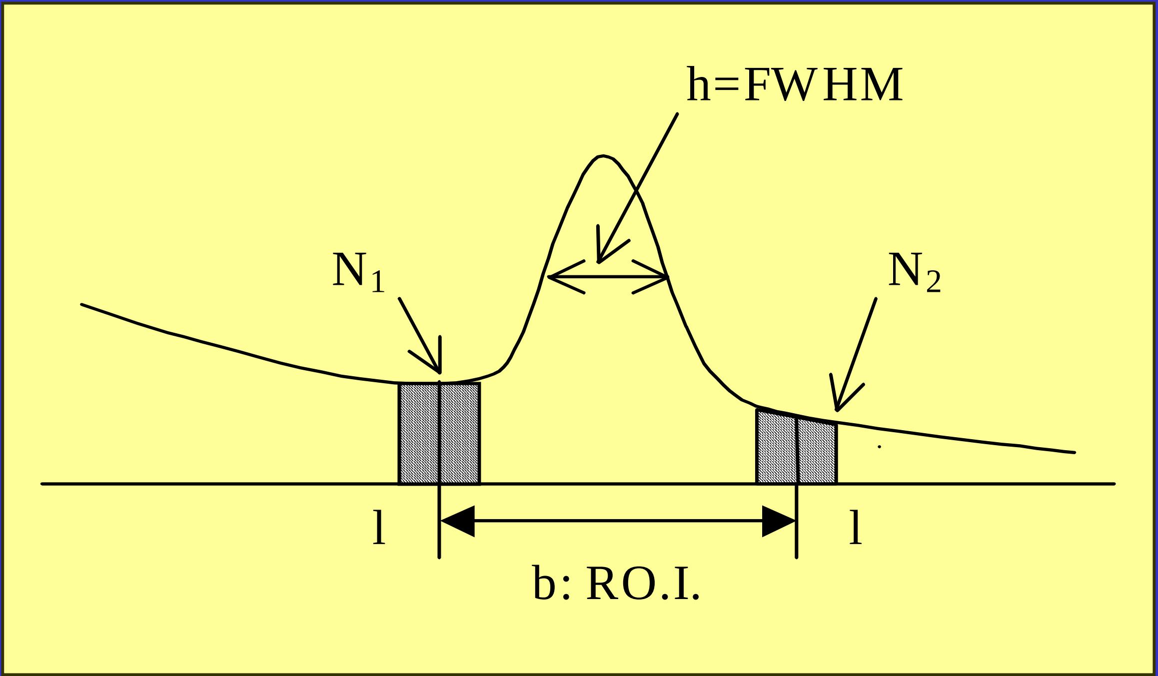
Estimation of background repeatability

a) Assume Poisson (or other) statistics and use the uncertainty propagation law (spectrometric measurements)

b) Measure the background variability if sources of fluctuation else than counting statistics are envisaged (sample treatment, counting system instability, environmental conditions …)

Example: Decision Threshold and Detection Limit in gamma-ray spectrometry

	Counts	Count. time	Count rate	Expectation value	Standard deviation
Background	N_0	t	R_0	υ_0	σ_0
Gross peak area	N_s	t	R_s	υ_s	σ_s
Net peak area	$N_n = N_s - N_0$	t	$R_n = R_s - R_0$	$\upsilon_n = \upsilon_s - \upsilon_0$	σ_n



$$N_n = N_s - N_0$$

$$N_0 = (N_1 - N_2)\frac{b}{2l}$$

$$b \geq 4 channels$$

$$b \leq 2l \leq 10b$$

$$N_n = N_s - (N_1 + N_2)\frac{b}{2l}$$

$$\mathrm{var}(N_n) = N_s + \frac{b}{2l}N_0$$

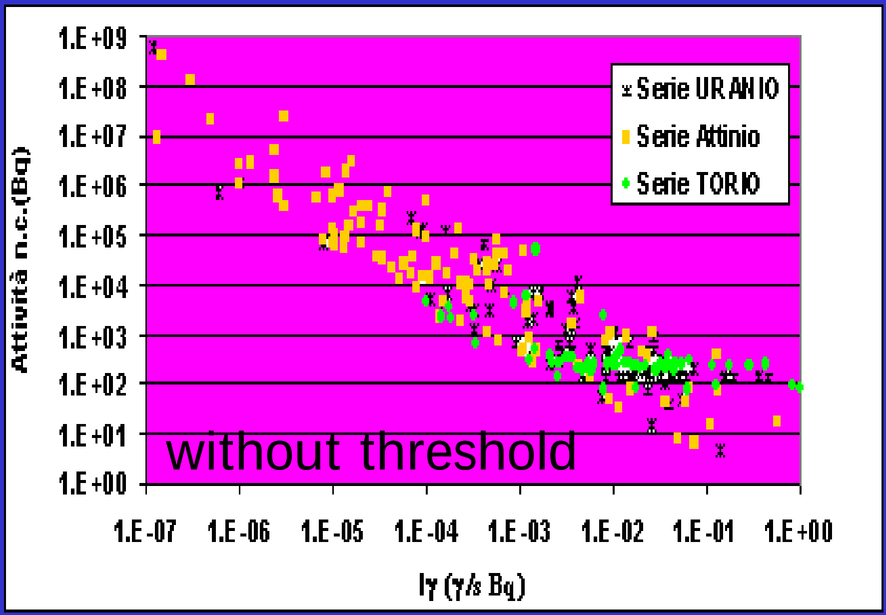
$$\mathrm{var}(\rho_n)\Big|_{\rho_n=0} = \frac{N_0}{t^2}(1+\frac{b}{2l})$$

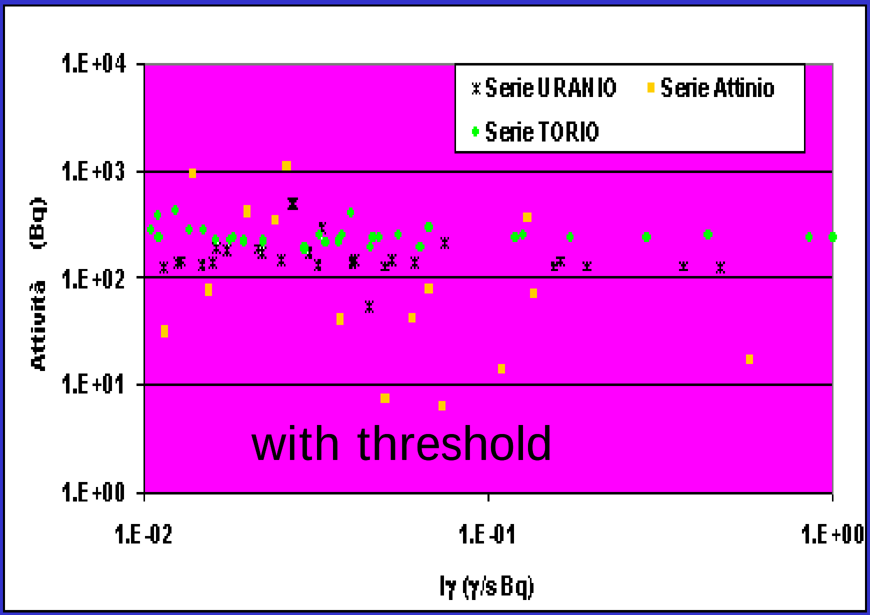
$$R_n^* = k_{1-\alpha}\sqrt{\frac{R_0}{t}(1+\frac{b}{2l})}$$

$$\rho_n^* = (k_{1-\alpha} + k_{1-b})\sqrt{\frac{R_0}{t}(1+\frac{b}{2l})}$$

$$1 < 1 + \frac{b}{2l} < 2$$

Example of application of a decision threshold





Thank you