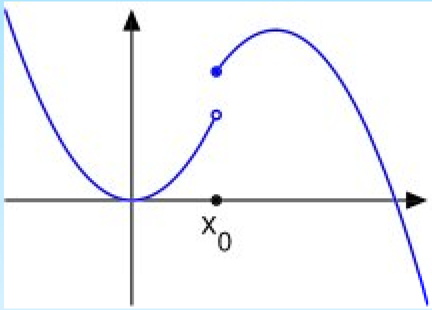
Efficiency calibration using experimental approach

Example of PTB (Fitting using Spline functions)

(By applying the program BSPLINE written by Herbert Janßen)

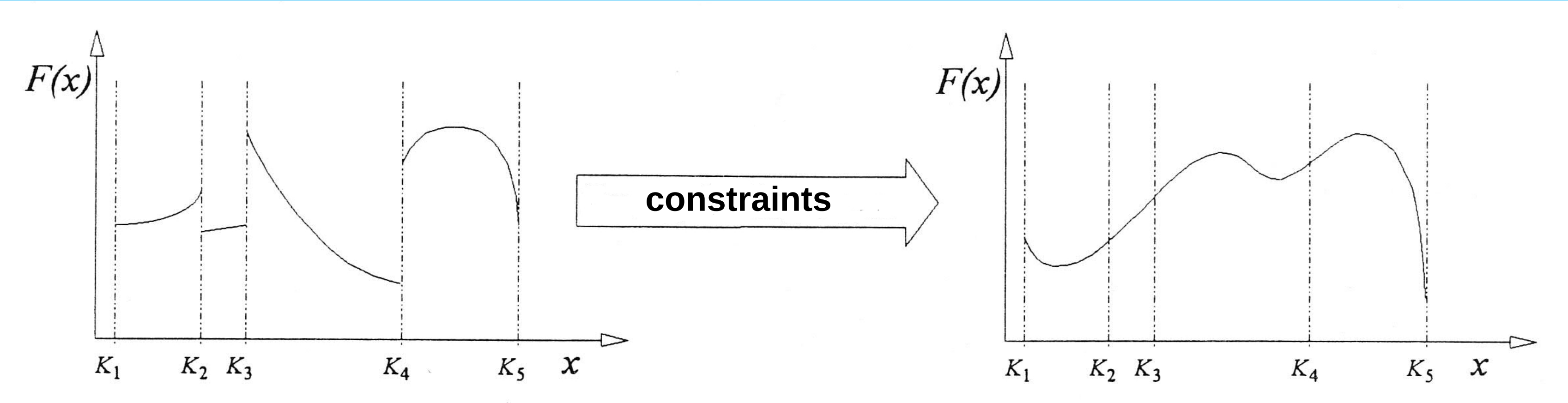
Spline functions

- Simply a curve
- Special function defined piecewise by polynomials



- A piecewise polynomial f(x) is obtainted by dividing of X into contiguous intervals, and representing f(x) by a separate polynomial in each interval
- The polynomials are joined together at the interval endpoints (knots) in such a way that a certain degree of smoothness of the resulting function is guaranteed

Piecewise defined polynomials



$\{K_i\}$ (i = 1, …, k+1)

$$K_1 \leq x_{min} < K_2 < \ldots < K_k < x_{max} < K_{k+1}$$

Series of knots

x_{min}: minimum x-value of the data points

x_{max}: maximum x-value of the data points

- 2 consecutive knots establish an interval $[K_i, K_{i+1})$ where a polynomial $P_i(x)$ of **order n** (degree n-1) is defined.
- In total k intervals are fixed by the knots.
- Outside the interval i the polynomial $P_i(x)$ is not defined.
- At the joining points $K_2 \ldots K_k$ they have to fit smoothly – **continous up to the (n-2) derivative**

Spline functions mathematically

k polynomials in k intervals: $F(x) = \sum_{i=1}^{k} P_i(x)$

where

$$P(x) = \begin{cases} \sum_{j=1}^{n} a_{ij} \cdot (x - K_i)^{j-1} & \text{for } K_i \leq x \leq K_{i+1} \\ 0 & \text{else} \end{cases}$$

Condition that a spline function solution must satisfy:

For the derivatons of neighboured polynomials at all knots:

$$P_i^{(m)}(K_{i+1}) - P_{i+1}^{(m)}(K_{i+1}) = 0, \quad 0 \leq m \leq n-2$$

BSPLINE:	Least square approximation of the data
	Minimize the sum of square distances to the curve

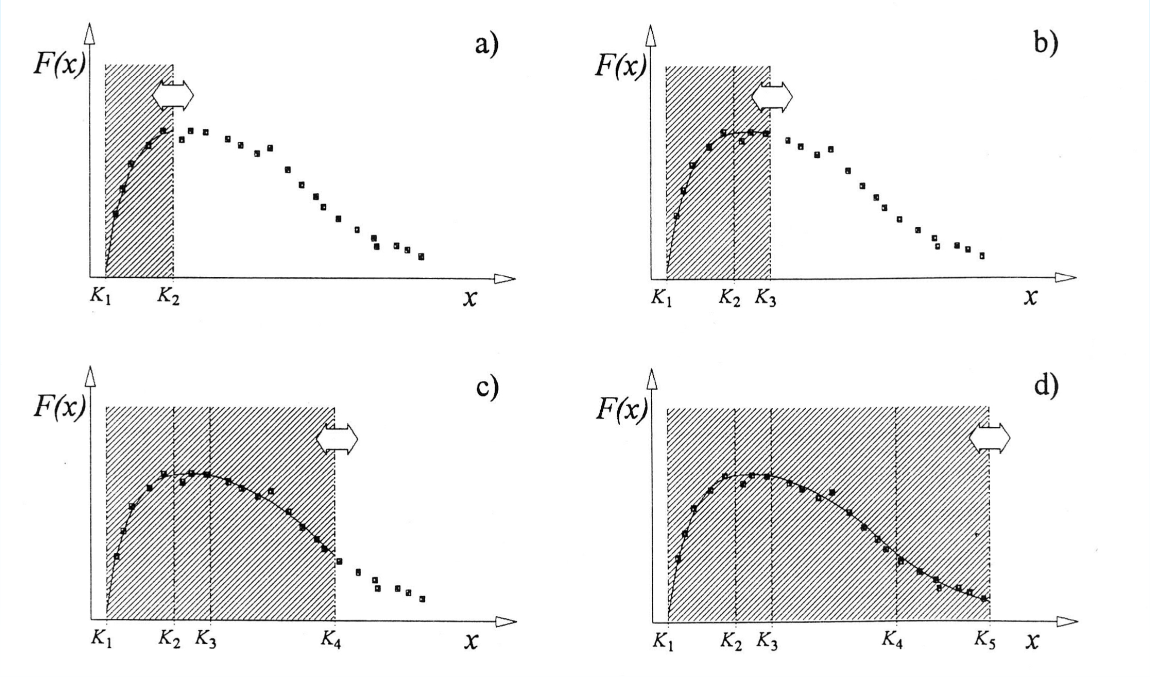
Choose the position of the knots

Trend indicator: $$T_i = \sqrt{\frac{p_i}{2}} \cdot \frac{R_i}{S_i} \qquad R_i = \sum_j r_{j-1} \cdot r_j \qquad S_i = \sum_j r_j^2 \qquad r_j = \frac{y_i - F(x_j)}{s_{y_j}}$$

P_i : number of data points in the interval
j: datapoints within an interval

The trend indicator is a measure for systematic variations in the residuals in case the functions are unsuitable. This can be for example if we have chosen an unsuitable series of knots or a wrong order

If there are no systematic variations (no trend) then $|T_i| < 1$

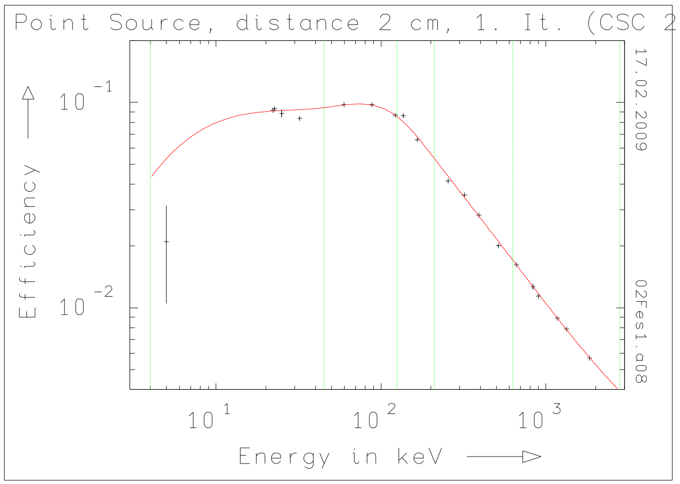


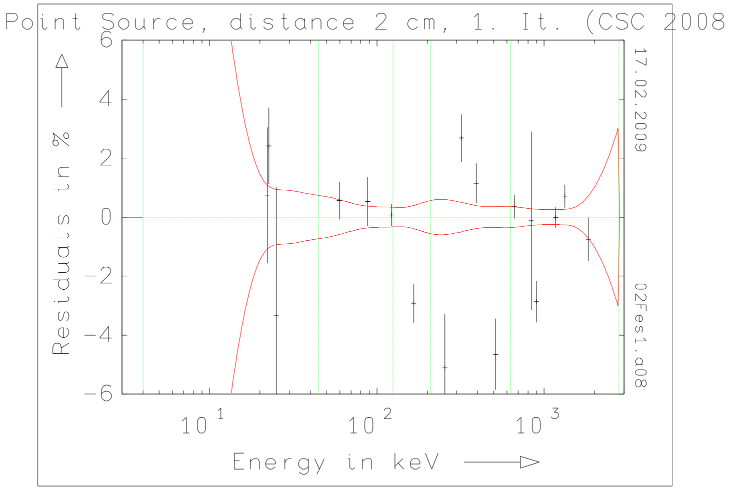
Typical BSPLINE control file

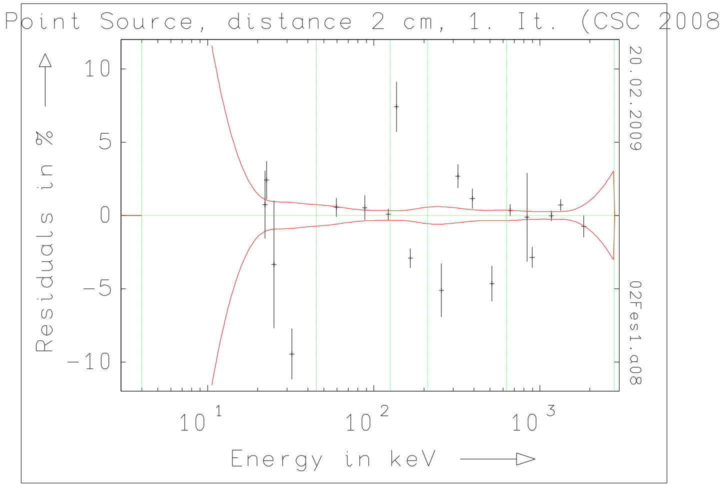
```
;Datei BS02B.a08 (Ablaufdatei für BSPLINE), 07.11.2008
clear data
clear nodes
clear parameters
set input triplets
restore data 02Fes1.a08
insert nodes 4 45 125 210 630 2800 >

set display 10203 = = = 3 3000 4E-3 2E-1 >
set stepsize 2
set fitmode loglog
set weightingmode instrumental
set errorlevel 1
fit bspline 4 >
display data
 set plot_param HP-GL "Eff02_1.hgl" >
  plot
;pause
 set plot_param HP-GL "Res02_1.hgl" >
 set display 10202 = = =  3 3000 -6 +6 >
 dis residuals
 plot
;pause
;EXIT
```

Efficiency 1. iteration

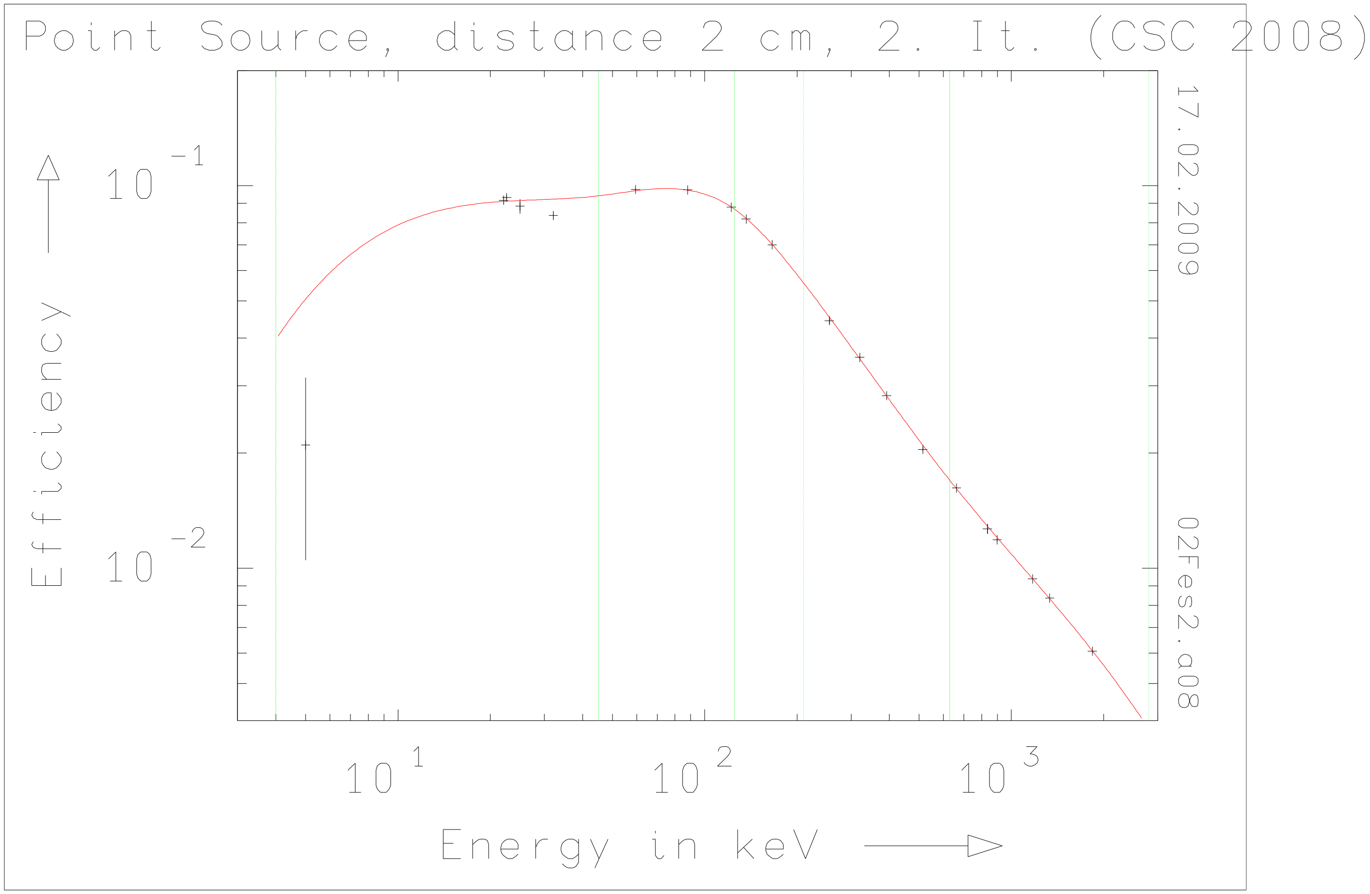




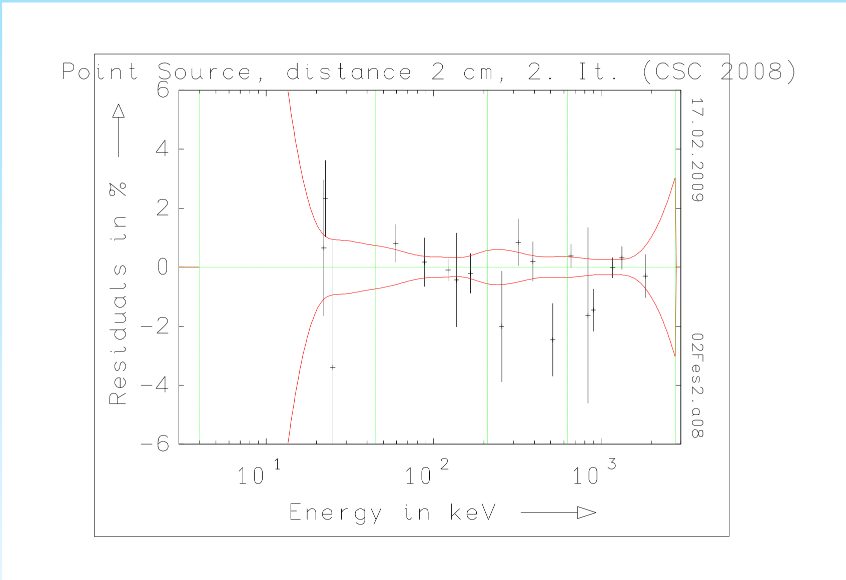


- peak area analysis at energies below 22 keV is not possible
- But: Efficienvy data for the K-Lines for various nuclides in the spectrum are necessary for coincidence summing correction for E>80 keV
- 5 keV data point estimated from the 22keV/5 keV ratio of similar PTB detectors
- Small error because it is only input for the calculation of the correction factor (KORSUM)

Efficiency 2. iteration



Residuals distance 2cm after 2. iteration



Red line: Standard deviation calculated by BSPLINE program

Uncertainty estimation: u(Eff) = 1% for the energy within the data points region

Reason: about 70 % of the input data are in the region ± 1% after the 2. iteration