**TITRE :** Characterization of a high purity germanium detector using SOLEX

**RESUME :**
La source monochromatique accordable SOLEX a été utilisée pour caractériser un détecteur GeHP commercial. Les principaux points étudiés sont l’homogénéité de la surface active du cristal de germanium ainsi que la fonction réponse du détecteur, plus particulièrement dans la gamme d’énergie 1-15keV, en particulier autour de la discontinuité d’absorption K du germanium. Cette première utilisation de SOLEX pour la caractérisation des détecteurs permet de mettre en avant l’intérêt de SOLEX en tant qu’outil métrologique.
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Introduction

High purity germanium (HPGe) detectors have been commonly used in gamma-ray spectrometry for many years. Up-to-now, their use in the X-ray energy range is less frequent, especially for energies lower than 20 keV, as the germanium dead layer is usually too thick to enable low-energy photons to reach the crystal active volume. Nevertheless, recent manufacturing improvements have reduced this drawback and some new HPGe detectors are designed to be used in the low-energy range. However, their use in a number of applications requires precise calibration of the semiconductor detectors. Their energy resolution and the detailed shape of their response function are parameters of interest for accurate processing of low-energy X-ray spectra to be applied to elements identification and fundamental research studies.

Such characterization has been performed for different semiconductor detectors (Si(Li), HPGe and SSD (silicon drift detector)) by means of a monochromatic radiation in the 1 - 20 keV energy range [Ref 1, Ref 2, Ref 3]. These first studies were conducted by the Laboratoire National Henri Becquerel (LNHB) at the LURE synchrotron facility. It was thus demonstrated that the use of a monochromatic photon beam, with tunable energy and intensity, allows accurate analysis of the detectors characteristics. The SOLEX (Source Of Low-Energy X-rays) source have been installed in late 2001 at LNHB for the purpose of providing tunable monochromatic radiation in the 1-20 keV energy range and then giving the French metrology institute the capability of developing detectors calibrations and metrological measurements in this energy range.

The present study conducted during the first three months of 2005, is the first example of SOLEX application to detectors characterization. It has been applied to a commercial HPGe detector, in the frame of a co-operation with Canberra. The study focuses on the examination of the homogeneity of the detector surface by means of the SOLEX scanning facility, and on the detailed characterization of the detector response function that is performed around the germanium binding energy.

1. Detector general characteristics

1.1 Commercial characteristics

The “Ultra Low Energy Germanium” detector provided by Canberra (model GUL0035) is presented in Figure 1; it consists in a N-type germanium crystal with 6.2 mm diameter and 5 mm thickness. It is housed under vacuum in a stainless steel casing. The beryllium window is 25 µm thick and the crystal is situated at 5 mm from it.
Figure 1: Picture of the studied HPGe detector with its electronic module

The detector is polarized under -500 V bias voltage. The nominative full width at half maximum (FWHM) is 136 eV at 5.9 keV using a 12 µs amplifier time constant.

1.2 Electronic module

For the present study, the detector is connected to a digital signal processing (DSP) module: DSPECplus™ including high voltage supply, pulses amplification and quasi-trapezoidal shaping based on digital processing with different options for the time constants. Pole zero compensation and base line restoration are automatic. The output signal is coded and sent to the MAESTRO® acquisition software that stores the energy spectrum. Using optimized shaping setups with a large time constant (rise time = 23 µs) the measured peak FWHM at 5.9 keV ($^{55}$Fe source) is 128 eV (Figure 2).

![55Fe spectrum](image)

Figure 2: Initial spectrum obtained with an $^{55}$Fe source to determine the FWHM at 5.9 keV
1.3 Energy and resolution calibration - Fano factor

First spectra are taken using radioactive sources ($^{55}$Fe, $^{57}$Co and $^{241}$Am) to perform energy and resolution calibration. The energy calibration is achieved in the 5-60 keV energy range and thus, it is possible to measure experimental energy resolution for different energies. The spectra are processed using the COLEGRAM software [See details in ANNEX 3: Peak processing]. The peaks corresponding to gamma ray emissions are fitted using Gaussian shapes and the X-ray peaks are fitted using Voigt shapes, corresponding to the convolution of the Lorentzian natural shape of emitted X-rays with the instrumental Gaussian widening.

The peak full width at half-maximum (FWHM) includes several components and increases with the energy [See A2-2 Peak width]. It is derived from the standard deviation of the Gaussian that is fitted to experimental data. These values are displayed in Figure 3 where the variance ($\sigma^2$) is plotted versus the energy.

Figure 3 : Plot of the square Gaussian standard deviation of the peaks versus the energy

As:

$$\text{FWHM}^2 = \Delta E^2 = K + \Delta E^2_s(E) = K + (2.355)^2 F \cdot w \cdot E$$  \[Eq. 1\]

And

$$\text{FWHM} = 2.355 \sigma$$  \[Eq. 2\]

Thus:

$$\sigma^2 = K + F \cdot w \cdot E$$  \[Eq. 3\]

w is the mean pair creation energy and F is the Fano factor.
The slope of the linear fitting to these experimental data is:

\[ 0.3148 = F \cdot w \]  

[Eq. 4]

With \( w = 2.96 \text{ eV} \) (for Ge at 77 K), this gives a Fano factor, \( F = 0.106 \). The constant part (electronic + collection defects) is \((1.269 \text{ eV})^2\), what represents about 84 eV in the peak FWHM.

1.4 Spectrum general features

Figure 4 displays the same spectrum as Figure 2, however with a longer acquisition time what allows to reveal more accurately the spectrum shape. It is obtained with a radioactive \(^{55}\text{Fe}\) source consisting of a drop of \(\text{FeCl}_2\) hydrochloric solution included between two mylar foils. The decay of \(^{55}\text{Fe}\) produces Mn K\(\alpha\) and K\(\beta\) X-rays with respective energies 5.895 keV and 6.490 keV, and respective natural Lorentzian width, \(\Gamma\), 1.5 and 2.8 eV. Using the logarithmic scale clearly shows the low-energy tail of both main peaks. A secondary peak at 2.620 keV is due to K fluorescence of chlorine contained in the hydrochloric solution.

The background shape also shows different features and discontinuities that are due to electron and photon escapes [See ANNEX 2: Resulting features in the X-ray spectrum]. They can be more easily detailed when using a monochromatic source.

The peak-to-background (defined as the ratio of the 5.895 keV peak intensity to the mean channel content in the 1 keV region) is 4,000.

The peak-to-valley (between the two peaks) ratio is: 25,000/55, that is 455.

\[ 55\text{Fe spectrum} \]

![Figure 4: Spectrum of Mn K X-rays obtained with a \(^{55}\text{Fe}\) source](image-url)
2. SOLEX source

2.1 General characteristics

The SOLEX (Source Of Low-Energy X-rays) source is a specific setup conceived to provide monochromatic radiation in the 1-20 keV energy range [Ref 4]. It basically includes an X-ray tube and a curved-crystal wavelength dispersive spectrometer both installed on an accurate mechanical system that insures that the monochromatic radiation is produced in a fixed direction. The spectrometer can be switched to either the Johann (reflection mode) or Cauchois (transmission mode) geometry by a simple rotation of the crystal. The whole setup is included in a large circular vacuum chamber. Figure 5 presents the scheme of the SOLEX mechanism.

The monochromatic output beam intensity depends on several parameters: X-Ray tube high voltage and current, crystal reflectivity, collimation and selected energy. The emission spectrum of the X-ray tube consists in a continuous spectrum (Bremsstrahlung) whose maximum energy (keV) correspond to the applied high voltage (kV) and anode characteristic X-rays whose intensity is much higher than the Bremsstrahlung one. Thus, the counting rate on the detector ranges between a few tens of photons per second, when the energy is selected in the Bremsstrahlung continuum, and several thousands per second when the energy selection is performed at the characteristic X-ray energy of the anode X-ray tube (example: copper Kα X-ray at 8.048 keV from the anode tube).

Due to its geometry (small length of the detector barrel), the studied detector cannot be installed directly inside the SOLEX source, thus, the monochromatic X-ray is used after its path through a 100 µm-thick beryllium window. Thus, it is not possible to use photons with energies lower than 3.5 keV, due to the attenuation of the incident beam in the beryllium window. The maximum energy is 12 keV, what allows analysing the germanium K binding energy.

![Diagram of SOLEX mechanism](image)

Figure 5 : Top view of the mechanism in the reflection mode. θ is the Bragg angle. C is the centre of the crystal; the vertical axis in C is the rotating axis of the crystal.
2.2 Crystal selection and spectral quality of the resulting photons beam

Different kinds of crystal (beryl, lithium fluoride with different cutting orientations, indium antimonide) are currently used to perform the energy selection from the continuous photon beam provided at the X-ray tube exit. The 3.5 to 12 keV energy range can be obtained using only the LiF(200) crystal in the reflection mode, however, the higher the energy, the lower the Bragg angle (14°) and scattered photons can directly reach the detector and then spoil the spectrum obtained with the monochromatic beam. Thus a new complementary configuration is used for the higher energy range: a quartz crystal is installed to work in the transmission mode. This allows using larger Bragg angles what should give more convenient experimental conditions.

2.2.1 LiF(200) crystal in the reflection mode

2.2.1.1 Collimating setup and resulting output photon flux

The mechanical setup used to define the exit beam geometry is presented in Figure 6. The external collimating system in front of the detector allows optimizing the diameter of the exit beam according to the required study. Therefore, the useful monochromatic beam has diameter between 350 µm and some mm. To determine the homogeneity of the detector active area, a small collimator is selected. To characterize the detector spectral response, a larger collimator can be used to get spectra with high statistical counting.
2.2.1.2 Spectral purity

Figure 7 displays the spectrum obtained with 12 keV monochromatic photons produced with the LiF(200) crystal in the reflection mode. The full-energy peak (12 keV) and both Kα, Kβ and L germanium escape peaks are clearly shown. The high voltage of the X-ray tube is 20 keV thus producing initial photons with 20 keV maximum energy, and maximum intensity flux around 13 keV. However, the resulting background is very low: its amplitude relative to the one of the main peak is less than 0.03 %, on either side of the peak. At the right side of the main peak, two pile-up peaks (main peak + Ge K escape peaks) appear.
2.2.2 Quartz crystal (0001) in the transmission mode

2.2.2.1 Collimating setup and resulting output photon flux

The study of the detector response function around the germanium K edge was performed with monochromatic beam produced from the quartz crystal in the transmission mode. In this configuration, the initial beam produced by the X-ray tube goes through the crystal, thus the spectrometer reflectivity is less that the one obtained in the reflection mode. The resulting photon flux is lower than the one obtained with the LiF crystal. It thus requires widening of the external collimation in order to illuminate the whole detection surface and also screening of the scattered photons inside the vacuum chamber.

Figure 8 displays the scheme of the relevant geometrical setup.
2.2.2 Spectral purity

With the transmission mode, the resulting beam purity is not as good as the one reached in the reflection mode.

Figure 9 presents a spectrum with 12 keV photons produced with such configuration. This can be directly compared with figure 7: in both spectra, the main and germanium escape peaks are comparable, however, in the last case, from the low energies to 20 keV maximum energy, the background is significantly higher. The background amplitude on the left side of the main peak is 0.06% relative to the one of the main peak, i.e. twice higher than when using the LiF(200) crystal. This enhanced background is due to scattered photons from the X-ray tube through the dispersive crystal. There are also larger zinc and copper K fluorescence peaks probably due to the crystal holder.

These features need to be pointed out, because they could constitute a drawback in the accurate analysis of the spectrum shape to study the peak tailing or quantification of low intensity characteristic (fluorescence or escape) peaks.
3. Scanning of the detector surface

For an optimum use of the detector, it is important to check the quantitative response of the different parts of the detector active surface to accurately define the zone with isotropic response. Indeed, peripheral areas could lead to degraded signals and then should be screened by a collimator.

To study this homogeneity of the detector active surface, the SOLEX source exit is equipped with a small collimator (diameter = 250 µm) and the detector is moved in front of it so that the incident beam reaches only a part of the detector active surface. The detector is fixed on an automated platform moving along both axes in a plane normal to the incident photon beam provided by SOLEX. This facility allows moving the detector with steps of 10 µm in front of the beam along axis X and Z. It is then possible to record the counting rate for each position, and also to store the relevant energy spectrum. The moving and acquisition sequences are completely automated and driven using software developed under LabView®. The scheme of the experimental arrangement is presented in Figure 10.

This experimental setup is used to check the homogeneity of the detection area by performing two series of scans. The energy selection is 8.048 keV and 5.0 keV, the lower the energy, the lower the interaction depth inside the germanium crystal: 90% of 8 keV photons are absorbed in a thickness of 66 µm of germanium, and for 5 keV, the relevant thickness is only 17 µm [See AI-3. Absorption coefficients – mean path]. Thus, the homogeneity is more accurately demonstrated using low energies.

Figure 9 : Spectrum obtained with 12 keV photons from the quartz in transmission mode
3.1 Scan at 8.048 keV

3.1.1 Beam dimension

The first measurement is performed using 8.048 keV monochromatic beam. This energy corresponds to copper Kα X-ray emission: as the energy selection is performed in the characteristic emission of the copper anode, the resulting flux is very intense and a small collimator can be used: The resulting diameter of the circular spot on the detector area is 350 µm, and the counting rate on the detector is around 1000 s⁻¹.

3.1.2 Experimental result

The scan is performed with 250 µm steps in both X and Z directions. For each position the counting rate on the detector (spectrum total area) is recorded and Figure 11 displays these experimental results.

The detector surface shows a homogeneous profile (measurement uncertainty around 1 %) and the measured active surface is a circle with about 5.5 mm-diameter.
3.2 Scan at 5 keV

3.2.1 Beam dimension

At 5 keV, the energy selection is performed in the Bremsstrahlung continuum, thus the resulting monochromatic beam has a lower flux than at 8.048 keV. It is thus necessary to open the collimation slits to send a larger part of the beam towards the detector. The resulting spot diameter is about 600 µm, giving about 350 pulses per second on the detector.

3.2.2 Experimental result

With these experimental conditions, the scan is performed with 500 µm steps in both directions. Figure 12 shows the experimental results, expressed as the total counting rate recorded by the detector at each scanning position. The measurement uncertainty is higher (about 3 %) than in the previous conditions, however, as the relative standard deviation of the experimental results is about 2.6 %, the detector homogeneity is still demonstrated.
4. Detector response function

Low-energy X-ray spectra obtained with semiconductor detectors reveal different features that need to be identified and quantified for accurate processing in a number of applications. These components are either due to the emission spectrum, such as radiative Auger effect and satellite lines, or to interactions in and around the detector crystal. Spectra obtained with incident monochromatic beam allows knowing the intrinsic “detector response function” versus the energy, i.e. the elementary component corresponding to interactions of photons with a single energy in the detector. This information can therefore be used to identify the different features of more complex spectra, thus providing improved processing results.

4.1 Energy range

In a first step, the 3.5 keV to 12 keV energy range is studied using the LiF(200) crystal. The monochromatic beam geometry is collimated to define a spot whose size is between 1 mm$^2$ and 2 mm$^2$, depending on the counting rate (the lower the energy, the larger the spot). As the detector homogeneity has been demonstrated, the beam is incident only on a part of the detector. In the studied energy range, spectra are taken with 500 eV steps, using acquisition times sufficient enough to record at least $5 \times 10^5$ pulses in the full-energy peak for each spectrum for the lower energies, and $10^6$ for the higher ones. It is important to get high counting statistics for an accurate analysis of the peak shape.

The energy range around the germanium K binding energy (11.103 keV) is of particular interest for the detector response function study. Thus, in a second step, a more detailed study is performed using the quartz crystal, in two series. In the first one, the energies are selected in the 11 keV to 11.2 keV range, using 20 eV steps. The second energy range is between 10.94 keV and 11.30 keV with 30 eV step. In each case, the acquisition time is adapted to reach good statistics (full-energy peak area containing more than $4 \times 10^5$ pulses).
4.2 Peaks processing

Spectra are processed using the COLEGRAM software [See ANNEX 3: Peak processing]. Figure 13 shows the full-energy peak region for 3.5 keV incident photon beam. Here, the full-energy peak has a Gaussian shape with low-energy tailing. In the upper part of Figure 13, the peak is fitted using a simple Gaussian shape. This function is well-adapted for fitting the peak central part, but this processing reveals severe residuals at the left side of the peak. In the lower part of Figure 13, the peak is fitted using the so-called “tailed Gaussian” function including a left-hand tail added to the basic Gaussian. The display of residuals in the lower part of the figure shows that the last processing mode is more appropriate for fitting the present peak shape.

Figure 13: Examples of spectrum processing at 3.5 keV
4.3 Experimental results

4.3.1 General peak shape

The general spectrum shape (peak shape and background intensity) is changing with the energy, as displayed in Figure 14 a-c. Moreover, above the germanium K binding energy, the germanium escape peaks appear. As demonstrated above, the tailed Gaussian shape is convenient for the experimental peaks fitting. It was thus used to process the spectra obtained with monochromatic photons in the 3.5 to 12 keV energy range. The fitting results give the values of the shape parameters that evolve versus the energy. The fitting parameters of interest are the Gaussian standard deviation ($\sigma$), tail slope and relative amplitude. The study of their variations allows determining some intrinsic characteristics of the detector that are developed hereafter.

![Graph showing count rates versus energy for different energies.](image-url)
4.3.2 Peak FWHM

A main characteristic of the peak is its Gaussian standard deviation, \( \sigma \), and the relevant FWHM (\( = 2.355 \sigma \)). The use of the SOLEX source can produce electronic noise in the detector electronics, depending on the source running conditions (tube high-voltage and current) thus widening the peak FWHM by increasing the value of the electronic noise contribution. As the tube alimentation conditions have to be adapted to optimize the measurement conditions (time and counting rate) over the whole energy range, this does not allow an absolute measurement of the peak FWHM. However, for a set of measurement with the same alimentation conditions, it is possible to study the evolution of the Gaussian standard deviation, \( \sigma \). Figure 15 presents the corresponding results (\( \sigma^2 \) versus the energy) obtained with constant conditions (tube high-voltage = 10.5 kV and current = 100 mA)

Here, the fitting of linear function to the experimental data gives:

\[
0.2966 = F \cdot w \quad [\text{Eq. 5}]
\]

With \( w = 2.96 \text{ eV} \), the relevant Fano factor, \( F = 0.100 \), what is consistent with the resolution measurement using radionuclides [See 1.3 Energy and resolution calibration - Fano factor].

On the other hand, as the constant part is \( (2 \text{ 146 eV})^2 \), the noise due to electronic and charge collection represents 109 eV in the peak FWHM, what is significantly higher than the one determined with the radionuclides (84 eV). This is mainly due to the presence of high voltage supply not far from the detector preamplifier, thus increasing the electronic noise.
4.3.3 Tail amplitude and slope versus the energy

Thus, the peak FWHM can depend on the energy due to external noise sources, but the peak shape that is due to the detector intrinsic characteristics does not change. The fitting parameters are recorded for each energy selection. The evolution of the peak tail amplitude is presented in Figure 16. It monotonically decreases from the lower energies to 11 keV, and then there is a sharp increase. Figure 17 displays the evolution of the tail slope: it also shows the same tendency with a decrease of the slope (i.e. reduction of the tail spreading) up to 10 keV then a discontinuity with an increase for higher energies.
4.3.4 Germanium escape peaks

4.3.4.1 L escape

Germanium L escape peaks appear in the low energy spectra, 1.2 keV below the full-energy peak and is superimposed on the tail. The escape peak shape is fitted using a Gaussian shape whose standard deviation is higher that the one of the main peak because the L escape “peak” actually consists in a group of peaks. Figure 18 presents an example of the processing including this feature.
The L escape intensity is very low (less than 0.2 %) relative to the full-energy peak intensity, and it decreases with the energy as displayed in Figure 19.

![Figure 19](image_url)

Figure 19: Intensity of the Ge L escape peaks group relative to those of the main peak (%)

### 4.3.4.1 K escape

Germanium K escape peaks appear when the incident energy is above the germanium K binding energy, i.e. 11.103 keV. Figure 20 shows the successive effects when energy selection is tuned between 10 keV and 12 keV.

![Figure 20](image_url)

Figure 20: Successive apparition of K escape peaks
Simultaneously with the K escape peaks, background on the right side of the escape peaks increases, due to escape of germanium K Auger electrons.

Remark : Together with the escape effect, pile-up appear on the right side of the main peak.

Figure 21 presents the intensity of escape peaks relative to the one of the full-energy peak measured with monochromatic radiation using two different crystals, and with radionuclides. These experimental values are compared with two series of computed values (continuous lines): the blue one results from analytical formula [See Annex 2 §3.1] and the pink one has been obtained with a Monte Carlo simulation [Ref 17]. Experimental results and Monte Carlo simulation agree. It can be remarked that the analytical formula gives significantly lower results, probably due to the use of simplified hypothesis.

Figure 21 : Relative intensity of the escape peaks (K₂α + K₂β) in the 12 to 40 keV energy range

A more detailed study focusing around the germanium K edge is presented in Figure 22.
4.4 Partially active layer

4.4.1 Tail-to-peak relative area

As above-mentioned, each peak includes a left-side tail whose relative intensity evolves versus the energy. Figure 23 presents the evolution of its area relative to the one of the full-energy peak.

Figure 22 : Relative intensity of the escape peaks above Ge K edge

Figure 23 : Relative intensity of the tail in the 3.5-12 keV energy range
The tail relative amplitude, that is about 10% at low energy, monotonically decreases as the energy increases. It dramatically increases above the germanium K binding energy to reach about 3%, value similar to the one obtained around 5 keV.

![Figure 24: Germanium linear attenuation coefficients in the 3.5–12 keV energy range](image)

This can be directly compared with the germanium linear absorption coefficient variation for the same energy range (Figure 24). As the absorption coefficient increases, the photons path inside the germanium is shorter: at 11 keV, the interaction depth corresponding to 90% photons absorption is 150 µm, and it is only 25 µm for 11.2 keV, just above the discontinuity. Photons thus interact near the entrance surface of the detector and thus, incomplete charges pair collection in this area increase the tailing amplitude (see § 4.2.2).

### 4.4.2 Partial active layer thickness

In the photon-germanium interaction around 11 keV, the initial energy is transferred to one primary electron (photo-electron or Auger electron) the energy and path of which are around 10 keV and 1 µm, respectively.

Along this path the primary electron creates secondary charges finally responsible for the output electrical pulse. If part of these charges is not collected, the electrical signal is weaker what gives a signal in the tail as displayed in Figure 25.

The partially active layer (PAL) is defined as this germanium zone where the charge collection is not complete, due to surface proximity what gives badly defined electrical field.
The tail relative intensity and its variation allow deriving the PAL thickness. Interactions taking place in the totally active layer give a signal in the full-energy peak (FEP), but the pulses resulting of interactions in the PAL appear in the tail. Thus, the ratio of the tail-to-peak areas corresponds to the probability of interaction in the PAL. The “peak” area is defined as the FEP including the tail + escapes peaks.

The transmission probability for a photon with initial energy \( E \) through the PAL is:

\[
P_{\text{Trans}}(E) = \exp \left( -\mu_{\text{Ge}}(E) \cdot \rho_{\text{Ge}} \cdot t_{\text{PAL}} \right) \quad \text{[Eq. 6]}
\]

Where:

\( \mu_{\text{Ge}}(E) \) is the germanium mass attenuation coefficient for energy \( E \) (cm\(^2\).g\(^{-1}\))

\( \rho_{\text{Ge}} \) is germanium density (5.32 g.cm\(^{-3}\) at 20 C)

\( t_{\text{PAL}} \) is the PAL thickness (cm)

Thus, the interaction probability in the PAL is:

\[
P_{\text{PAL}}(E) = 1 - P_{\text{Trans}}(E) = 1 - \exp \left( -\mu_{\text{Ge}}(E) \cdot \rho_{\text{Ge}} \cdot t_{\text{PAL}} \right) \quad \text{[Eq. 7]}
\]

Thus:

\[
t_{\text{PAL}} = -\ln \left[ 1 - P_{\text{PAL}}(E) \right] \bigg/ \mu_{\text{Ge}}(E) \cdot \rho_{\text{Ge}} \quad \text{[Eq. 8]}
\]
Figure 26 shows the experimental PAL thickness for different energies in the 3.5-12 keV range, derived from [Eq. 9], using mass attenuation coefficients from XCOM [Ref 8]. Very consistent values are obtained, except just around the germanium K binding energy where fine absorption structures occur and give discrepant results. When removing these values (in the 11.06 – 11.12 keV range), the mean PAL thickness is 280(10) nm.

### 4.5 Efficiency

The whole intrinsic efficiency calibration could not been performed as there was no reference detector for monitoring the monochromatic beam intensity at the time of the present experimental study. However, some Monte Carlo simulations have been performed in order to get efficiency values. The PENELOPE code [Ref 17] ran for the HPGe detector with dimensions such as described in the manufacturer specifications, including a 10 nm-thick germanium dead layer and 100 nm-thick aluminium layer, for a point source located at 1 cm from the beryllium window.

The results of the simulation as displayed in Figure 27 include both the total and full-energy peak efficiencies over the 2-40 keV energy range. Examples of resulting spectra are given in Figure 28 for incident photons below and above the germanium K binding energy. Raw data (deposited energy) are displayed in the upper panel and resulting spectrum (including the detector Gaussian widening) are displayed in the lower panel. These last spectra can be more easily compared with the experimental ones shown in Figure 20.
Figure 27: Results of Monte Carlo simulation for detector efficiencies for a point source at 1 cm from the detector entrance window.

Figure 28: Simulated spectra obtained with incident photons at 10.5 and 11.5 keV. Upper panel: raw data (deposited energy) – lower panel: data including Gaussian widening.
Conclusion

The present study was the first attempt of use of SOLEX for detector characterization. The goal has been attained. The detector homogeneity was demonstrated and the different features included in the experimental spectra could be explained.

This experiment was also the opportunity to test in details the functioning conditions of SOLEX for such kind of experiment and to determine the relevant measurement procedures. This also gave the opportunity to make some improvements in the installation, particularly concerning the vacuum quality and the ability to get photon energies up to 30 keV.

After its use in different applications concerning the measurement of attenuation coefficients [Ref 18, Ref 19], the present study demonstrates that SOLEX is an excellent equipment for metrological purposes.

In this step, only qualitative characterization of the detector efficiency was available. However, SOLEX was recently equipped with a new platform allowing installing a second detector that can be used in parallel with the detector to be characterized. With this monitoring system, it will be possible to include a reference detector, such as a proportional counter. With this future equipment, it will possible to obtain quantitative results and perform absolute efficiency calibration of detectors for LNHB demands and also for external users.
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Ref 9: F.B. Larkins, At. Data and Nucl. Data Tables 20 (1977) 313


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ANNEX 1: Photon interactions with matter

In the X-ray energy range, the main interactions of photons with matter are photoelectric effect and Compton scattering, both inducing secondary effects.

**A1-1 Primary interactions**

**A1-1.1 Photoelectric effect**

In the photoelectric effect, the photon energy (E) is totally transferred to an electron of an inner atomic shell. This results in the ejection of the inner electron, so-called “photoelectron” (pe), with energy:

\[ E_{pe} = E - E_i \]  

[Eq. 10]

E is the energy of the initial X-rays. 
E\(_i\) is the binding energy of the electron on its initial electronic shell.

For germanium the internal electronic shell concerned are only K and L:

The binding energy of the K shell is [Ref 9]:

\[ E_K = \text{11.1031 keV} \]

The L shells include 3 sub-shells with respective binding energies [Ref 9]:

\[ E_{L1} = \text{1.4143 keV} \]
\[ E_{L2} = \text{1.2478 keV} \]
\[ E_{L3} = \text{1.2167 keV} \]

Remark: the photoelectric effect probability strongly increases as soon as the incident energy exceeds any atomic shell binding energy, because the corresponding shell can be in its turn ionized.
**A1-1.2 Compton scattering**

The Compton scattering is an interaction between the incident photon, with energy $E$, and a peripheral electron, that is ejected (Compton electron). In this process, only part of the initial energy is transferred to the Compton electron and the remaining energy appears as a secondary photon. The energy sharing depends on the angle, $\theta$, between the Compton electron and the scattered photon.

![Compton scattering diagram](image)

**Figure 30 : Compton scattering**

The energy, $E'$, of the scattered photon is:

$$E' = \frac{E}{1 + \frac{E}{m_0 c^2 (1 - \cos \theta)}} \quad [\text{Eq. 11}]$$

And the Compton electron carries the remaining available energy:

$$E_{ce} = E - E' = E \left[ 1 - \frac{1}{1 + \frac{E}{m_0 c^2 (1 - \cos \theta)}} \right] \quad [\text{Eq. 12}]$$

Remark: in fact, the really available energy for sharing between scattered photon and Compton electron is the initial energy minus the energy required to eject the electron. However, as the Compton scattering mainly occurs at medium and high energies, the energy of the external atomic shell is weak in front of the incoming photon energy, and is often neglected in the energy budget.

**A1-2. Electronic rearrangement**

In both initial interactions, the ejection of the electron creates a vacancy in the atomic shell, thus leaves the atom in an ionized state. In the photoelectric effect, the vacancy is created in
an inner-shell and it is filled by an electron coming from a less bound electronic shell. This rearrangement induces either emission of X-ray or Auger electron [Ref 10].

**A1-2.1 Characteristic X-rays**

In the radiative process, the rearrangement energy is released as a secondary photon whose energy is characteristic of the absorbing material, as it depends on the binding energy corresponding to the initial and final vacancies.

A vacancy in a K shell can be filled by electrons from a L shell (K$\alpha$ rays) or M shells (K$\beta$ rays).

![Figure 31: Emission of characteristic X-ray](image)

The resulting energy of characteristic KX-rays is the energy of the initial vacancy minus the energy of the final vacancy. For an initial vacancy in the germanium K shell, the characteristic K X-rays of germanium are:

$$E_X = E_i - E_f \quad [\text{Eq. 13}]$$

$$E_{XK\alpha_1} = E_K - E_{L2} = (11.103 - 1.248) \text{ keV} = 9.855 \text{ keV}$$
$$E_{XK\alpha_2} = E_K - E_{L3} = (11.103 - 1.217) \text{ keV} = 9.886 \text{ keV}$$

$$E_{XK\beta_1} = E_K - E_{M3} = (11.103 - 0.121) \text{ keV} = 10.982 \text{ keV}$$
$$E_{XK\beta_2} = E_K - E_{N2-N3} = (11.103 - 0.0023) \text{ keV} = 11.101 \text{ keV}$$

The L X-rays are consecutive to an initial ionization in a L shell and the main mean energies are:

$$E_{XL\alpha} = 1.188 \text{ keV}$$
$$E_{XL\beta} = 1.224 \text{ keV}$$
$$E_{XL\gamma} = 1.412 \text{ keV}$$

**A1-2.2 Auger electron**

In the Auger effect, the rearrangement energy is transferred to an external electron (Auger electron) that is ejected and carries out the excess energy.
Figure 32: Emission of an Auger electron

In this radiationless three steps process, the Auger electron has the energy of the initial vacancy (here, K) minus the sum of the energy of the two external shells where new vacancies are created (here, L and M for example).

For germanium, an initial vacancy in a K shell gives one K Auger electron, among a series of possible combinations, depending on the external shells concerned by the cascade rearrangement:

For example, K-L2-L3:

\[ E_{\text{Auger}} = E_K - E_{L2} - E_{L3} = (11.103 - 1.248 - 1.217) \text{ keV} = 8.638 \text{ keV} \]

Or K-L2-M3:

\[ E_{\text{Auger}} = E_K - E_{L2} - E_{M3} = (11.103 - 1.248 - 0.121) \text{ keV} = 9.734 \text{ keV} \]

The L Auger electron energies in germanium range between 0.0697 keV and 1.3806 keV, 0.0261 keV and 1.2141 keV, and 0.8567 keV and 1.183 keV for an initial vacancy in the L1, L2 and L3 subshells, respectively [Ref 11].

Remark: Coster-Kronig transitions are the particular cases of an Auger electron emission where the vacancy in the subshell \( X_i \) is filled by an electron from a higher subshell \( X_j \), in the same major shell \( X \), while the ejected electron may come from the same or a higher major shell.
<table>
<thead>
<tr>
<th>Auger electrons number for 100 vacancies in the K shell</th>
<th>Energy (keV)</th>
<th>Mean energy (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KL1L1</td>
<td>8.2745</td>
<td>8.5639</td>
</tr>
<tr>
<td>KL1L2</td>
<td>8.441</td>
<td>8.6075</td>
</tr>
<tr>
<td>KL1L3</td>
<td>8.4721</td>
<td>8.6386</td>
</tr>
<tr>
<td>KL2L2</td>
<td>8.6075</td>
<td>8.6697</td>
</tr>
<tr>
<td>KL2L3</td>
<td>8.6386</td>
<td>8.6966</td>
</tr>
<tr>
<td>KL3L1</td>
<td>8.441</td>
<td>8.4721</td>
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<tr>
<td>KL1L2</td>
<td>8.441</td>
<td>8.4721</td>
</tr>
<tr>
<td>KL1M1</td>
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<td>KL1M2</td>
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<tr>
<td>KL1M3</td>
<td>9.568</td>
<td>9.6601</td>
</tr>
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<td>KL1M4,5</td>
<td>9.6601</td>
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</tr>
<tr>
<td>KL2M1</td>
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<td>KL2M2</td>
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<td>KL2M4,5</td>
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<td>KL3M4,5</td>
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<td>10.7431</td>
<td>0.046</td>
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<td>KM1M2</td>
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<td>0.039</td>
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<tr>
<td>KM1M3</td>
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<td>KM2M3</td>
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<tr>
<td>KM3M3</td>
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</tr>
<tr>
<td>KM3M4,5</td>
<td>10.9536</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Table 1: K Auger electron energies and number for 100 K vacancies in germanium [Ref 11]

**A1-2.3 Radiative Auger effect**

There are also secondary rearrangement phenomena; where there is a mixing between photon emission and Auger electron emission; it is called the radiative Auger effect and the energy is then shared between the two particles [Ref 12, Ref 13]. The resulting photons thus have lower energies than characteristic X-ray photons: this induces complex features in the low energy side of the full-energy peak.

**A1-2.4 Fluorescence yield**

The fluorescence yield \( \omega_I \) of an atomic shell, I, is the probability that a vacancy in the corresponding I shell be filled by the radiative process (X-ray emission). Thus, the fluorescence yield is the number of photons emitted when a vacancy is filled divided by the number of primary vacancies.

The Auger yield, \( a_I \), is the probability that a vacancy in the I shell is filled through a non radiative transition.

\[
\omega_I + a_I = 1 \quad \text{[Eq. 14]}
\]

For Germanium [Ref 14]:

Page 36 sur 51
K = 0.535
L1 = 0.0024
L2 = 0.0013
L3 = 0.0015

Thus, for a vacancy in the L shell, there is a much higher probability of Auger electron emission than X-ray emission.

**AI-3. Absorption coefficients – mean path**

The attenuation of a mono-energetic photon beam with initial energy E with normal incidence through any material follows the Beer-Lambert law:

\[ I = I_0 \exp(-\mu X) \]  

[Eq. 15]

\[ I_0 \] is the initial beam intensity,

\[ I \] is the transmitted beam intensity,

\[ X \] is the thickness of the absorbing material

\[ \mu \] is the total linear attenuation coefficient of the material for energy E.

The total attenuation coefficient is the sum of partial attenuation coefficients corresponding to the elementary interactions (photoelectric effect (τ), Compton scattering (σ), Pair production (κ)). In the X-ray range, as there is only photoelectric and Compton effects, this gives:

\[ \mu = \tau + \sigma. \]

The photoelectric absorption coefficient is the sum of partial coefficients corresponding to the photoelectric effect with each electronic sub-shell. \[ \tau = \tau_K + \tau_{L1} + \tau_{L2} + \tau_{L3} + \tau_{M1} + \tau_{M2}... \]

This effect strongly increases as soon as the incident energy is higher than any binding energy, as in these case, the photoelectric effect also occurs in the relevant shell.

Table 2 displays the partial interaction coefficients of germanium versus the energy [Ref 8] including the discontinuities at the L and K binding energies.
<table>
<thead>
<tr>
<th>Photon Energy (MeV)</th>
<th>Scattering (Compton) (cm²/g)</th>
<th>Photoelectric Absorption (cm²/g)</th>
<th>Total Attenuation</th>
<th>Ratio Compton/photo</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00E-03</td>
<td>5.34E+00</td>
<td>6.19E-03</td>
<td>1.89E+03</td>
<td>1.89E+03</td>
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<td>1.10E-03</td>
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<td>1.19E+03</td>
<td>1.19E+03</td>
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Table 2 Partial attenuation coefficients of germanium [Ref 8]

As above-mentioned, for low energies, the photoelectric effect dominates: at 10 keV, the ratio Compton-to-photoelectric attenuation coefficients (last column) is less than 0.2 % and up to 50 eV, the Compton effect probability is less than 5 % relative to the photoelectric one.
Thus, 50 % of the beam will be attenuated if:

\[ I = \frac{I_0}{2} = I_0 \exp(-\mu x) \rightarrow X = -\ln(0.5)/\mu \]  
[Eq. 16]

And 90 % will be attenuated for:

\[ X = -\ln(0.1)/\mu \]  
[Eq. 17]

Here, the material of interest is the germanium crystal, thus, using the corresponding attenuation coefficient, it is possible to derive the path of the incident beam (for example, 50 % and 90 % attenuations) in the detector versus the energy what allows to approximate the first interaction location.

10 keV : 50 % : 35 µm      90 % : 116 µm
15 keV : 50 % : 14 µm      90 % : 47 µm

As the mass absorption coefficients are given in cm$^2$.g$^{-1}$, the germanium specific mass is included, $\rho_{\text{Ge}} = 5.323$ g.cm$^{-3}$

As a comparison,
Table 3 presents the electron path for energies ranging from 10 to 20 keV.

<table>
<thead>
<tr>
<th>Energy(keV)</th>
<th>Path (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.92</td>
</tr>
<tr>
<td>12.5</td>
<td>1.33</td>
</tr>
<tr>
<td>15</td>
<td>1.80</td>
</tr>
<tr>
<td>17.5</td>
<td>2.33</td>
</tr>
<tr>
<td>20</td>
<td>2.92</td>
</tr>
</tbody>
</table>

Table 3: Stopping power for electrons [Ref 15]
ANNEX 2: Resulting features in the X-ray spectrum

A2-1 Full-energy peak

If the energy of the incident photon, $E$, is totally absorbed into the detector crystal through consecutive effects, the electrical signal is proportional to $E$ thus giving in the energy spectrum the full-energy peak (FEP): it is a Gaussian-like distribution centred on $E$. This main feature is accompanied by secondary features due to the different escape possibilities:

Remark: the radiative Auger effect gives photons with lower energy, thus appearing in the low-energy side of the main peak [Ref 12, Ref 13].

A2-2 Peak width

According to its interaction in the detector and electronics, this line with energy $E$ will give a peak with finite width in the final spectrum. Three main factors influence the peak width:
- Statistics of the charge-creation process,
- Collection charge,
- Electronic noise.

When a photon interacts in the active crystal, either by photoelectric effect or Compton scattering, its initial energy is transferred to one (or more) electron(s) that loses its (their) energy mainly by ionization, finally resulting in electron-hole pairs creation. The energy requested to create a pair is $w = 2.96$ eV in germanium at 77 K. Thus, the number of charge carriers pairs created is: $N = E/w$.

This creation pair process is described by a standard deviation, $\sigma_S$ expressed as $(F.N)^{1/2}$, where $F$ is the Fano factor. This factor is an intrinsic property of the semiconductor material. The resulting standard deviation expressed in energy is:

$$\sigma_S = \sqrt{F \cdot E \cdot w} \quad [\text{Eq. 18}]$$

and the corresponding statistical Gaussian width is:

$$\Delta E_S(E) = 2.355 \sigma_S \quad [\text{Eq. 19}]$$
The two other contributions, due to incomplete charge collection and electronic noise are independent on the energy (in a first approximation). The incomplete collection of the created charge can be due to partial or total trapping of a charge carrier: consequently, the resulting electrical pulse is smaller than expected and is coded in a channel lower than the one corresponding to full-energy absorption. This effect mainly contributes to low energy tailing of the peak. The electronic system induces also noise that widens the peaks. This contribution depends primarily on the leakage current of the detector and its capacitance; it can be more or less influenced by the quality of the electronic chain and its adaptation to the initial pulse shape.

Finally, all these contributions add quadratically and the squared full width at half maximum (FWHM) of the resulting peak is:

\[
\Delta E^2 = \Delta E_s^2(E) + \Delta E_c^2 + \Delta E_E^2 \quad \text{[Eq. 20]}
\]

Where \(\Delta E_s\) is statistical width, \(\Delta E_c\) is charge collection width and \(\Delta E_E\) is electronic noise width:

\[
\Delta E_s(E) = 2.355 (F \cdot w \cdot E)^{1/2} \quad \text{[Eq. 21]}
\]

Thus, a first approximation of FWHM expressed versus the energy can be written:

\[
\Delta E = \left[ K + \Delta E_s^2(E) \right]^{1/2} = \left[ K + (2.355)^2 F \cdot w \cdot E \right]^{1/2} \quad \text{[Eq. 22]}
\]

### A2-3 Escape processes

#### A2-3.1 Photons escape

As the initial photon interaction induces secondary processes, the secondary particles in turn will either interact within the detector crystal or escape from it.

A characteristic X-ray will interact mainly by photoelectric effect, thus depositing its energy inside the detector. However, if the initial interaction takes place near any edge of the crystal, the rearrangement photon can escape from the crystal active volume. Finally, the deposited energy is the incident photon energy minus the energy of the characteristic X-ray of the detector (here, germanium):

\[
E_{K\text{esc}} = E - E_{Kx} \quad \text{(If E is higher than the germanium K binding energy)} \quad \text{[Eq. 23]}
\]

For germanium, as there are 2 groups of characteristics X-rays, the spectra show two K escape peaks:

- \(E_{K\alpha\text{esc}} = E - 9.9 \text{ keV}\)
- \(E_{K\beta\text{esc}} = E - 11 \text{ keV}\)

And L X-ray escape gives a “peak” with energy:

\[
E_{L\text{esc}} = E - E_{Lx} \quad \text{(If E is higher than binding energy of L-shell).} \quad \text{[Eq. 24]}
\]

\(E_{L\text{esc}} = E - 1.2 \text{ keV}\)
Thus, escape of characteristic X-rays will give escape peaks with Gaussian shapes signal in the left side of each full energy peak.

![Figure 34: Photon escape peaks](image)

The intensity of the escape peaks depends on the initial photons energy: it can be calculated using certain assumptions [Ref 16]. For normal photon incidence and neglecting escape through the sides and rear of the detector, the ratio of the number of counts in the K escape peak, $N_e$, to the number of counts in the full-energy peak, $N$, is:

$$\frac{N_e}{N} = 0.5 \omega_K \left( 1 - \frac{1}{r} \right) \left[ 1 - \frac{\mu(E_K)}{\mu(E)} \cdot \ln \left( 1 + \frac{\mu(E)}{\mu(E_K)} \right) \right]$$

[Eq. 25]

$E$ is the energy of incident photons
$E_K$ is the mean energy of germanium K X-rays
$\omega_K$ is the germanium K-shell fluorescence yield = 0.535
$r$ is the ratio of germanium attenuation coefficients just above and before the K edge
$r = 197/28.1 = 7.011$
$\mu$ is the germanium attenuation coefficient at $E$ or $E_K$.

The ratio can be computed for both K escape peaks ($\alpha$ and $\beta$) using the relevant energies and relative intensities. These theoretical ratios are presented in Figure 35.
A2-3.2 Electrons escape

A2-4.3.1 Photoelectron escape

Contrary to photons that lose their energy through well-defined interactions, the electrons lose continuously their energy along their path. If one electron escapes from the detector active volume, it will take only part of its initial energy, that is:

\[ E_{pe} = E - E_b \]  \hspace{1cm} [Eq. 26]

The escape of the photoelectron will correspond to loss of energy ranging between zero and the maximum (initial) energy of the photoelectron, \( E_{pe} \).

Finally, this effect results in a continuous tail, with energy \( E_{esc} \) between \( E_b \) and \( E \) (Figure 36).
Then, if $E$ is higher than the germanium K binding energy, the photoelectron escape continuum starts at 11.103 keV, and for lower energies, the L photoelectron escape continuum starts at the mean L binding energy, i.e. 1.2 keV.

**A2-4.3.2 Auger electron escape:**

In the same way, the escape of an Auger electron will correspond to loss of energy ranging between zero and the maximum (initial) energy of the Auger electron $E_A$.

This phenomenon will give a signal in a continuum with energy $E_{esc}$ starting at $E - E_{Auger}$ till the full-energy peak, $E$.

![Figure 37: Auger electron escape continuum](image)

For germanium as the mean K Auger electron energies are:

- $E_A(KLL)= 8.56$ keV
- $E_A(KLM)= 9.70$ keV
- $E_A(KMM)= 10.83$ keV

The Auger electron escape continuum starts around $E - 10$ keV if $E$ is higher than the germanium K binding energy, and around $E - 1.2$ keV for lower incident energies.

Remark: this continuum starts in the same energy region as the escape peak.

**A2-4 Features due to interaction outside the germanium**

If the preliminary interaction occurs outside of the germanium active part, the secondary particles can in turn enter the detector crystal and leave their energy, thus producing features in the spectrum.

**A2-4.1 Fluorescence photons**

If characteristic X-rays created outside the germanium penetrate into the detector, they will deposit their energy and create fluorescence peak(s).
A2-4.2 Secondary electrons

In the same way, secondary electrons can also finally arrive inside the germanium crystal, however with a degraded energy: this induces continuous features with energies $E_{\text{inp}}$ ranging from 0 to the initial energy of the secondary (Photoelectron or Auger electron) electron.
ANNEX 3: Peak processing

Spectra are processed using the COLEGRAM software [Ref 6]. This code is used to fit a mathematical function to a set of experimental data. Different fitting functions are available, depending of the peak shape.

A3-1 Gaussian shape

The simplest and most commonly used is the Gaussian shape, \( G(E) \) that is characterized by three parameters: energy, \( E_0 \), amplitude \( A \), and standard deviation \( \sigma \).

\[
G(E) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{(E-E_0)^2}{2\sigma^2} \right] 
\]

[Eq. 27]

A3-2 Tailed Gaussian shape

The so-called “tailed Gaussian” function includes a left-hand tail, \( T(E) \), added to the basic Gaussian. The tail is modelled as an exponential function truncated at the peak position, convolved with a Gaussian representing the instrumental broadening:

\[
T(E) = \int_{-\infty}^{E_0} A \cdot \exp(\tau E') \cdot \exp \left[ -\frac{(E'-E_0)^2}{2\sigma^2} \right] \cdot dE' 
\]

[Eq. 28]

what can also be expressed in a simpler way for computing:

\[
T(E) = A \cdot \frac{T}{2} \cdot \exp \left[ (E-E_0)\tau + \frac{\sigma^2 \tau^2}{2} \right] \cdot \text{erfc} \left[ \frac{1}{\sqrt{2}} \left( \frac{(E-E_0)}{\sigma} + \sigma \tau \right) \right] 
\]

[Eq. 29]

This shape includes two more parameters: tail relative amplitude \( T \), and exponential slope \( \tau \).

A3-3 Voigt shape

The Voigt function is adapted to the shape of X-ray peaks as it is the result of convolving a Lorentzian function (corresponding to the natural shape of X-ray line) with a Gaussian profile taking account of the instrumental broadening:

\[
V(E) = \int_{-\infty}^{\infty} L(E') \cdot G(E-E') \cdot dE' 
\]

[Eq. 30]

Where:

\[
L(E) = \frac{\Gamma/2\pi}{(E-E_0)^2 + (\Gamma/2)^2} 
\]

[Eq. 31]

To entirely describe the Voigt function, it is necessary to include the Lorentzian width, \( \Gamma \), in the parameters used for the Gaussian shape.
ANNEX 4: Theoretical efficiency

A1-1 Definitions

A1-1.1 Total efficiency

The detector total efficiency is the ratio of the number of counts in the spectrum to the number of photons emitted by the source.

\[ \varepsilon_T(E) = \frac{\Omega}{4\pi} \prod_{i} (\exp(-\mu_{i}(E) \cdot x_{i}) \cdot (1 - \exp(-\mu_{Ge}(E) \cdot x_{Ge}))) \quad [\text{Eq. 32}] \]

The integration must be conducted on the solid-angle sustained by source and the germanium active surface, \( \Omega \). The product is for the different absorbing layers (i) between the source and the active volume.

If the source-to-detector distance, \( d \), is large in front of the crystal radius, \( r \), (\( d/r > 10 \)), the beam incoming on the detector can be consider as parallel, thus [Eq. 32] simplifies:

\[ \varepsilon_T(E) \approx \frac{\Omega}{4\pi} \prod_{i} (\exp(-\mu_{i}(E) \cdot x_{i}) \cdot (1 - \exp(-\mu_{Ge}(E) \cdot x_{Ge}))) \quad [\text{Eq. 33}] \]

The intrinsic total efficiency of the detector can be defined as the total efficiency where solid angle and any absorbing layer before the detector window are not taken into account, thus:

\[ \varepsilon^I_T(E) = \prod_{i} (\exp(-\mu_{i}(E) \cdot x_{i}) \cdot (1 - \exp(-\mu_{Ge}(E) \cdot x_{Ge}))) \quad [\text{Eq. 34}] \]

A1-1.2 Full-energy peak efficiency

The full-energy peak efficiency, \( \varepsilon^P \), is the ratio of the number of counts in the full-energy peak (FEP) to the number of photons emitted by the source.
\[ \varepsilon^p(E) = \prod_{i} \left( \exp(-\mu_i(E) \cdot x_i) \right) \left( 1 - \exp(-\mu_{\text{Ge}}(E) \cdot x_{\text{Ge}}) \right) \cdot P_{\text{FEP}} \cdot F_{\text{esc}} \cdot d\Omega \]  

[Eq. 35]

This equation includes two complementary factors, \( P_{\text{FEP}} \) and \( F_{\text{esc}} \), respectively the probability of interaction by total absorption of the initial energy and the correction due to escape effect.

In the case where the incoming beam can be considered as parallel, and as in the low-energy range most of the interaction is photoelectric effect, and an approximated expression of \( \varepsilon^p(E) \) is:

\[ \varepsilon^p(E) \approx \frac{\Omega}{4\pi} \prod_{i} \left( \exp(-\mu_i(E) \cdot x_i) \cdot \left( 1 - \exp(-\tau_{\text{Ge}}(E) \cdot x_{\text{Ge}}) \right) \right) \cdot F_{\text{esc}}(E) \]  

[Eq. 36]

For intrinsically characterizing the detector FEP efficiency, solid angle and any absorbing layer before the detector window are not taken into account, thus:

\[ \varepsilon_i^p(E) = \prod_{i} \left( \exp(-\mu_i(E) \cdot x_i) \cdot \left( 1 - \exp(-\tau_{\text{Ge}}(E) \cdot x_{\text{Ge}}) \right) \right) \cdot F_{\text{esc}}(E) \]  

[Eq. 37]

Here, the known absorbing layers (i) are beryllium window (25 µm), aluminium (100 nm) and germanium dead layer (10 nm). Using the absorption coefficients from [Ref 8], the computation elements of the intrinsic total and full-energy peak efficiency are presented in Table 4 and the resulting curves are displayed in Figure 41.

---

Figure 41 : Theoretical HPGe intrinsic efficiency (pink = FEP, blue = total)
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Table 4: Theoretical HPGe detector efficiency