THE ATOMIC MASSES

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I - INTRODUCTION TO THE NUCLEUS – NUCLEAR BINDING ENERGY

II - EXPERIMENTAL METHODS FOR MEASURING MASSES

III - THE ATOMIC MASS EVALUATION

IV - THE ATOMIC MASS ADJUSTMENT PROGRAM

V - ESTIMATES OF UNKNOWN MASSES

VI - NUBASE

VII - The AMDC
DECAY MODES
- $\beta^+$ (EC + $e^+$)
- $\beta^-$
- $\alpha$
- Internal Transition
- Spontaneous Fission
- $p$
- n
- Stable nuclide
- Unknown decay
Découverte de $^{30}$P, premier noyau artificiel

1934 Paris

328 nucléides terrestres

2800 nucléides artificiels à prévoir avec les accélérateurs terrestres

2786 en 2000

Monique Bernier
march 2002
ISOMERS AND NUBASE

- **gs – isomer identification**
  - if $\gamma$-emission $\Rightarrow$ Nsdd
  - if no $\gamma \Rightarrow$ decay energy to other nuclide $\Rightarrow$ AME

- **NUBASE ‘horizontal’ evaluation**
  - half-lives
  - spin and parities
  - decay modes
  - which state is the ground-state
  - which states are involved in a mass relation

- **Other needs for NUBASE**
  - radioactive parameters for calculations
  - reactors
  - waste management
  - nuclear astrophysics
  - prepare nuclear physics experiment
Experimental Data

- **Reaction Energies** eV
  \[ Q_r = M_A + M_a - M_b - M_B \]
  - close to stability
  - \((n,\gamma)\) and \((p,\gamma)\) \Rightarrow backbone
  - self-calibrated \(A(a,b)B\) v/s \(C(a,b)D\)

- **Desintegration Energies** eV
  \[
  A(\beta^-)B \quad Q_{\beta^-} = M_A - M_B \\
  A(\alpha)B \quad Q_{\alpha} = M_A - M_B - M_\alpha \\
  A(p)B \quad Q_p = M_A - M_B - M_p
  \]
  - far from stability

- **Mass Spectrometry** u (often called “Direct”)
  1. Classical Spectrometers
  2. Time-of-Flight Spectrometers \((M^t (B\rho)\) and velocity)
  3. Cyclotron Spectrometers
     a. Radio-Frequency Spectrometer
     b. Penning Trap Spectrometer
     c. Storage Ring Spectrometer
Precision for $^{14}\text{N}$

One order of magnitude every 10 years

- 1935: 400 keV
- 1995: 0.9 eV
- 2003: 0.5 eV
Penning Trap: $463 \pm 7$ keV

Literature: $463.62 \pm 0.09$ keV

M. König 1991

NDS
The more recent history of atomic masses can be found in:
Georges Audi
“The history of nuclidic masses and of their evaluation”

An early (perhaps the first) attempt for a mass evaluation is
M.S. Livingston, H.A. Bethe, “Nuclear Physics, C. Nuclear dynamics, experimental”,
Rev. Mod. Phys. 9 (1937) 245, XVIII. Nuclear masses; p. 366
The authors combined data from mass spectrometry and nuclear reaction and
decay data up to 40Ar.

In the early 1950’s it was found that the many relations (direct and indirect)
overdetermined the mass value of many nuclides.
Aaldert H. Wapstra established a procedure using a least-squares method to
solve the problem of overdetermination.
The first table of atomic masses using this method is dated 1955.
History of Wapstra’s type AME


A.H. Wapstra & K. Bos, At. Data Nucl. Data Tables 19 (1977) 175


Evaluation of Nuclear Data

- **Theoretical evaluation** (＝ theoretical predictions)
  - predict values for large sets of nuclei
  - make effective calculations for: r-process - reactors - ...
  - uncertainties: just starting  security \( \leq \) feasibility

- **Evaluation of experimental data**
  - basic blocks: results from experiments
  - if available: prefered to theoretical predictions
  - allows also to test theoretical “evaluations”
Experimental Data

- Energy Relation
  
  expressed in $\text{eV}$ or $\text{keV}$

  $\text{keV}^*$: standard volt
  adopt a value for $2e/h$ in Josephson

  $\text{keV}$: international volt
  from evaluation of fundamental constants

- Inertial mass in EM field
  
  expressed in $u$ or $\mu u$, the
  “unified mass unit”

  $1u = M^{(\text{C})}/12$ since 1960

Conversion factor:

$1u = 931.494.009.0 \pm 0.007.1 \ \text{keV}^*$

$1u = 931.494.013 \pm 0.037 \ \text{keV}$
Special Treatments

- Asymmetric Errors $X^{+a}_{-b}$
  
  Symmetrize the probab. distribution:

  - Rough symmetrization
    \[ X + \frac{1}{2}(a - b) \pm \frac{1}{2}(a + b) \]
  
  - Rigorous symmetrization (cf. Nubase2003)
    \[ X + 0.64(a - b) \pm \sqrt{(1 - \frac{2}{\pi}) (a - b)^2 + ab} \]
Special Treatments

- **Range of Values** \( X_{min} - X_{Max} \)

Moments of the probab. distribution:

\[
\frac{1}{2}(X_{min} + X_{Max}) \pm 0.29(X_{Max} - X_{min})
\]
Special Treatments III-a

- **Mixture of two lines**

  Two lines are known to exist at: $M_{gs}$, $M_m$
  but relative population NOT known

  Assume equal probability for all population ratio
  The mixture will appear at any value between $M_{gs}$ and $M_m$

  \[ M_{exp} = \frac{1}{2}(M_{gs} + M_m) \pm 0.29(M_m - M_{gs}) \]

  the ground-state mass ($E_1 = M_m - M_{gs}$):

  \[ M_{gs} = M_{exp} - 0.5E_1 \pm \sqrt{(\sigma_{exp}^2 + (\frac{1}{2}\sigma_1)^2 + (0.29E_1)^2} \]
Special Treatments

- Mixture of three lines

Three lines are known to exist at: \( M_{gs} \quad M_{m1} \quad M_{m2} \)

(see demonstration in Ame2003, p. 176):

the ground-state mass:

\[ M_{gs} = M_{exp} - \frac{1}{3}(E_1 + E_2) \]

and its error:

\[ \sigma_{gs}^2 = \sigma_{exp}^2 + \left( \frac{1}{3}\sigma_1 \right)^2 + \left( \frac{1}{3}\sigma_2 \right)^2 + \frac{1}{18}(E_1^2 + E_2^2 - E_1 E_2) \]
LIFETIME = \[ \frac{T_{1/2}}{\ln 2} \]

lower limit = \[ \frac{T_{1/2}}{1 + \sqrt{n}} \]

negative error = \[ \frac{T_{1/2}}{1 + \frac{1}{\sqrt{n}}} \]

upper limit = \[ \frac{T_{1/2}}{1 - \sqrt{n}} \]

positive error = \[ \frac{T_{1/2}}{1 - \frac{1}{\sqrt{n}}} \]

<table>
<thead>
<tr>
<th>( n )</th>
<th>upper limit</th>
<th>posit. error</th>
<th>lower limit</th>
<th>neg. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.79</td>
<td>+4.79 \cdot T_{1/2}</td>
<td>0.543</td>
<td>-0.457 \cdot T_{1/2} \text{ exact}</td>
</tr>
<tr>
<td>2</td>
<td>2.82</td>
<td>+1.82</td>
<td>0.606</td>
<td>-0.394 \cdot T_{1/2} \text{ exact}</td>
</tr>
<tr>
<td>3</td>
<td>2.37</td>
<td>+1.37</td>
<td>0.634</td>
<td>-0.366</td>
</tr>
<tr>
<td>4</td>
<td>2.00</td>
<td>+1.00</td>
<td>0.667</td>
<td>-0.333</td>
</tr>
<tr>
<td>5</td>
<td>1.81</td>
<td>+0.809</td>
<td>0.691</td>
<td>-0.309</td>
</tr>
<tr>
<td>6</td>
<td>1.69</td>
<td>+0.649</td>
<td>0.710</td>
<td>-0.290</td>
</tr>
<tr>
<td>7</td>
<td>1.61</td>
<td>+0.608</td>
<td>0.726</td>
<td>-0.274</td>
</tr>
<tr>
<td>8</td>
<td>1.55</td>
<td>+0.547</td>
<td>0.739</td>
<td>-0.261</td>
</tr>
<tr>
<td>9</td>
<td>1.50</td>
<td>+0.500</td>
<td>0.750</td>
<td>-0.250</td>
</tr>
<tr>
<td>10</td>
<td>1.46</td>
<td>+0.462</td>
<td>0.760</td>
<td>-0.240</td>
</tr>
</tbody>
</table>

\( n = 1 \) \rightarrow \ T + 4.79 \cdot T \cdot 0.457

\( n = 2 \) \rightarrow \ T + 1.82 \cdot T \cdot 0.394
comparison of mass model predictions

MISTRAL Collaboration

CSNSM-Orsay
Mass Formulae Predicting Powers

* Mass Relations
  "Local Relations"
  "Systematics"

* Semi-Empirical Formulae
  Collective Model
  Independent Particle Model

  (Liquid Drop,...)
  (Shell Model → Liran & Zeldes)

* Fundamental Methods
  Nilsson’s Shell Model
  H.F. Methods

QUANTITATIVE COMPARISON:

Simple Mean (deviations - exp. error)

Current Methods:
- Mean Square deviation
- Weighted deviations
The Ame (“Atomic Mass Evaluation”)

- Regularity of the mass-surface

  Smoothness and structures
  - Surface in 3D space
  - Structures on the Surface: Shells - Deformations - Wigner

  Regularity is a basic property of the surface of masses

Consequences:
- New Physics
- Outliers
- Conflict among Data
- Estimate unknown masses
Regularity of the Mass Surface

- Surface in 3D space
- Pairing Energy $\Rightarrow$ 4-sheets
  - nearly parallel in all directions
  - smooth variations in $N$ and $Z$

  Caveat: smooth $=$ continuous non-staggering
  smooth $\neq$ slow

- Structures on the Surface:
  Shells - Deformations - Wigner

- Conclusion: Regularity is a Basic Property
Regularity of the Mass Surface II

- New Physics
  - Coherent deviations in \((N, Z)\)
    \[ \Rightarrow \text{new physical property (e.g. } ^{23}\text{N}_{15} N = 108 - 115 \text{ Cs}_{63-112} \) \]

- Outliers
  - One single ‘Irregularity’
    \[ \Rightarrow \text{question correctness of datum} \]
    \[ \text{re-measure same and/or measure neighbors} \]
    \[ \text{strongly deviating 1-experiment (chaotic surf.):} \]
    \[ \Rightarrow \text{replace by estimated ‘recommended’ value} \]

- Conflict among Data
  \[ \Rightarrow \text{which one agrees with estimate?} \]

- Unknown Masses
  \[ \Rightarrow \text{Estimates: Interpolate - Extrapolate} \]
Regularity of the Mass Surface

- **Extrapolations** (short extrapolations)
  
  from regularity of the surface of masses
  
  for medium and heavy nuclides
  
  consider several graphs

  knowledge of n-stable or n-unstable
  
  for light n-rich nuclides
  
  similar for p-rich ← but Coulomb !!

  mirror and IMME
  
  for light p-rich nuclides
Fig. 6. Mass Exp-Mass Duflo-Z 96 sph. $N = 70$ to 118
Recent observations of the mass surface

- Magic numbers vanishing (quenching) when going far from stability earlier: $N = 20, N = 28$
  now also (since Ame2003): $N = 50, N = 82$

- Rising of the wings $Q_\beta$’s often underestimated
  r-process paths less far from stability
  expected influence on reactors and on waste calculations
Fig. 2. Mass Exp-Mass Duflo-Z 96 sph. N= 8 to 46
Fig. 3. Mass Exp-Mass Duflo-Z 96 sph. N = 18 to 66
Fig. 4. Mass Exp-Mass Duflo-Z 96 sph. $N = 34$ to 86

Neutron Number $N$

Mass Exp-Mass Duflo-Z 96 sph. + Z*1.0 (MeV)

Isotopes shown:
- $^{71}$Rb, $^{73}$Sr, $^{76}$Y, $^{81}$Mo, $^{85}$Tc, $^{87}$Zr, $^{89}$Rh, $^{91}$Pd, $^{93}$Ag, $^{95}$Cd, $^{97}$In, $^{99}$Sn, $^{103}$Sb, $^{105}$Sr, $^{108}$Y, $^{110}$Zr, $^{113}$Nb, $^{115}$Mo, $^{117}$Cs, $^{118}$Tc, $^{120}$Ru, $^{122}$Rh, $^{124}$Pd, $^{126}$Ag, $^{130}$Ag, $^{132}$Cd, $^{135}$In, $^{136}$Sn, $^{137}$Sb, $^{139}$Sb, $^{141}$Sb, $^{147}$Sb, $^{150}$Sb, $^{151}$Sb
TO CONCLUDE

- Deriving a mass value from one or several experiments sometimes requires expertise

- Mathematical tools (LSM) + computer tools + evaluator’s judgment

  are essential ingredients to reach the best possible mass-values

- unknown masses
  - close to last ones: predicted from extension of mass surface
  - further out: derived from models.

  but models diverge!!! (10’s of MeV in region of r-process)

- therefore:
  - best possible experimental data
  - best possible evaluation of masses \rightarrow best set of mass values

  on which models may 1) adjust their parameters
  2) improve predictions further away
Nuclear Data

- **Static Nuclear Data**
  - masses
  - $T_{1/2}$ & $J^\pi$
  - excitation energies
  - decay modes at rest and intensities
  - decay from excited states
  - neutron capture cross-sections
  - magnetic moments & radii
  - ...

- **Dynamic Nuclear Data**
  - reaction cross-sections
  - reaction rates
  - reaction mechanisms
  - spectroscopic factors
  - ...
## "Static" Nuclear Data

<table>
<thead>
<tr>
<th>Z = 1</th>
<th>Z = 2</th>
<th>Z = 3</th>
<th>Z = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A = 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A = 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A = 4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**structures and decays**

(Ensdf)

(NSDD network)

**Ame**: created by A.H. Wapstra (Amsterdam) in 1959 since 1981 together with G. Audi (Csnsm – Orsay)

interation Ame $\leftrightarrow$ Ensdf

$\Rightarrow$ **NUBASE**: masses, isotopes $E^m$

$T_{1/2}$, $J\pi$

decay modes & intensities
“Horizontal Evaluations”

- **Atomic Mass Evaluation (AME)**
  Wapstra since 1959 + G.A. since 1981

- **NUBASE**
  Blachot, Bersillon, Wapstra, G.A. since 1993

- **Radii, spins and moments**

- **Isotopic abundances**
  Holden

- \( E_{2+} \) and \( B(E2) \) for e-e nuclides

- ...
The Nubase evaluation of Nuclear and Decay Properties

Ground state
Isomeric states, > 100 ns

- Masses
- $T_{1/2}$
- $J^\pi$
- Decay modes and intensities

<table>
<thead>
<tr>
<th>Element</th>
<th>Mass</th>
<th>$T_{1/2}$</th>
<th>$J^\pi$</th>
<th>Decay Mode</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>146 Dy</td>
<td>-62555</td>
<td>7</td>
<td>33.2 s 0.7 0+</td>
<td>97</td>
<td>93 TcO5d 81, B+=100</td>
</tr>
<tr>
<td>146 Dy</td>
<td>-59619</td>
<td>7</td>
<td>2935.7 0.6</td>
<td>150 ms 20</td>
<td>10+#</td>
</tr>
<tr>
<td>146 Ho</td>
<td>-51238</td>
<td>7</td>
<td>3.6 s 0.3 (10+)</td>
<td>97</td>
<td>82, B+=100; B+p=?</td>
</tr>
<tr>
<td>146 Ho</td>
<td>146Ho</td>
<td>E : AHW975: &quot;I tend to think 1+ 200#150 above (unknown) 1&quot;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>146 Ho</td>
<td>W146Ho</td>
<td>E : GAu978: not supported by syst.: no 1+/10+ comb. And c-o syst not good!!</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>146 Er</td>
<td>146Er</td>
<td>-44322</td>
<td>1.7 s 0.6 0+</td>
<td>97</td>
<td>93 TcO5d 93, B+=100; B+p=?</td>
</tr>
<tr>
<td>146 Tm</td>
<td>-146Tm</td>
<td>-31280#</td>
<td>400#</td>
<td>155 ms 20 (1+)</td>
<td>05, Rc40t 05, p-100; B+ ?; B+p ?</td>
</tr>
<tr>
<td>146 Tm</td>
<td>146Tm</td>
<td>T : also 05, Bb02=190 (80) ms</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>146 Tm</td>
<td>*146Tm</td>
<td>-31010#</td>
<td>400#</td>
<td>300 10 p</td>
<td>77.5 ms 2.4 (5-,5-)</td>
</tr>
<tr>
<td>146 Tm</td>
<td>146Tm</td>
<td>T : average 05, Bb02=82 (4) 05, Bb02=75 (3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>146 Tm</td>
<td>W146Tm</td>
<td>T : discovered by 93Li18 for gs and 146Tm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>146 Tm</td>
<td>W146Tm</td>
<td>T et J : Ensdif'97 gs=(10+) 235 ms 27 m=(5-,6-) 72 ms 23 &lt;= ok in Ensdif2002</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>146 Tm</td>
<td>*146Tm</td>
<td>-30930#</td>
<td>400#</td>
<td>475 30 p</td>
<td>213 ms 9 (10+,9+,8+)</td>
</tr>
<tr>
<td>146 Tm</td>
<td>146Tm</td>
<td>T : from 2007da2U</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>146 Tm</td>
<td>W146Tm</td>
<td>D : B+ estimated from calculated $T(\beta+C+B^+)=430$ ms</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

“The Nubase Evaluation of Nuclear and Decay Properties”
Nuclear Physics A729 (2003) 3
AMDC: the Atomic Mass Data Center
http://amdc.in2p3.fr/

- Mass Evaluations Ame’83, Ame’93, Ame’95 and Ame2003
texts, tables, figures of the publication + extra figs. and bonus
- Nubase Evaluation Nubase’97 and Nubase2003
- Preprints on masses:
  Experimental – Evaluation – Theory
- Other products:
  jvNubase – Nucleus PC-program – . . .
  + Ensdf-files + . . .

+ BULLETIN e-mail
  - 1000 addressees
  - activities related to masses
    Evaluation – Experimental – Theory

MEETING POINT and
EXCHANGES → MASSES

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