Towards a determination of the absorbed dose to water in water for x-rays below 50 keV

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Low-energy photon-emitting brachytherapy seeds

Permanent seeds e.g. for the treatment of prostate cancer:

\[ ^{125}\text{I}: \quad 27.20, 27.47 \text{ keV} \]
\[ \quad 31.00, 31.71 \text{ keV} \]

\[ ^{103}\text{Pd}: \quad 20.07, 20.22 \text{ keV} \]
\[ \quad 22.72, 23.18 \text{ keV} \]

\[ D_w \text{ in a water-phantom} \]
Overview

- Extrapolation chamber Set-up
- Problem: Extrapolation from SEE
- Extrapolation Method
- Comparison to a free air chamber
- Uncertainties: Conversion factors
- Results
- Conclusion
Graphite Extrapolation Chamber

\[ K_w^g (0) = K_{air}^g (0) \cdot \frac{(\mu_{en} / \rho)^w}{(\mu_{en} / \rho)_{air}} \]
Extrapolation from SEE

\[ D_{air}(x) = \left( \frac{W}{e} \right) \cdot \frac{1}{\rho_{air} A} \cdot \frac{dQ(x)}{dx} \]

\[ Q(x) \]

\[ \frac{Q(x_{i+1}) - Q(x_i)}{x_{i+1} - x_i} \]
• Beam divergence
• Stray radiation
• Absorption
Photon Fluence Spectra (FluRZnrc)
**Method I**

**SEE:** \( K_{\text{air}}^g(x) = D_{\text{air}}^g(x) \)

\[
Q(x) = \left( \frac{e}{W} \right) \rho_{\text{air}} A \cdot \overline{D}_{\text{air}}(x) \cdot x
\]

\[
k_{KC}^{MC}(x) = \frac{K_{\text{air}}^g(0)}{K_{\text{air}}^g(x)} = \frac{\int \Psi(E,0)^{MC} \cdot (\mu_{en}(E)/\rho)^{air} \cdot dE}{\int \Psi(E,x)^{MC} \cdot (\mu_{en}(E)/\rho)^{air} \cdot dE}
\]
Method II

\[
\int \int \cdot \cdot \Psi \cdot \cdot \Psi = \int \int dE_x K_x k \Psi_{air} \cdot E_{air} \cdot (\mu / \rho)_{air} (E) \cdot dE
\]

[Equation for absorbed dose/kerma]

[Diagram showing photon paths and absorbed dose/kerma]

\[
k_{MC} (x) = \frac{K_{MC} (0)}{K_{MC} (0)} = \int \Psi (E, x) \frac{MC}{MC} (E) \cdot (\mu / \rho)_{air} (E) \cdot dE
\]

[Equation for absorbed dose/kerma]

[Diagram showing photon paths and absorbed dose/kerma]
Method III

\[ k_{KC}^{MC} (x_{i+1}) \cdot Q(x_{i+1}) - k_{KC}^{MC} (x) \cdot Q(x) = \left( \frac{e}{W} \right) \rho_{air} A \cdot \left( k_{KC}^{MC} (x_{i+1}) \bar{K}^g_{air} (x_{i+1}) \cdot x_{i+1} - \right. \]

\[ k_{KC}^{MC} (x_i) \cdot \bar{K}^g_{air} (x_i) \cdot x_i \right) = \left( \frac{e}{W} \right) \rho_{air} A \cdot K^g_{air} (0) \cdot (x_{i+1} - x_i) \]

\[ K^g_{air} (0) = \frac{W}{e} \frac{1}{\rho_{air} A} \frac{k_{KC} (x_{i+1}) \cdot Q(x_{i+1}) - k_{KC} (x_i) \cdot Q(x_i)}{x_{i+1} - x_i} \]
Conversion factor \((x, E)\)

\[
k_{KC}(x) = \left( A_1 \exp\left(-\frac{x}{p_1}\right) + A_2 \exp\left(-\frac{x}{p_2}\right) + y_0 \right)^{-1}
\]
\[ Q(x) \]

\[ \frac{Q(x_{i+1}) - Q(x_i)}{x_{i+1} - x_i} \]
Linear Extrapolation
Comparison with a free air chamber
## Conversion factors

<table>
<thead>
<tr>
<th>Radiation Quality</th>
<th>$\frac{(\mu_{en}/\rho)^w}{(\mu_{en}/\rho)^{air}}$</th>
<th>$C_{\alpha,\alpha}^{w,g}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N 20</td>
<td>1.021 ± 0.8 %</td>
<td>0.4106 ± 2.1%</td>
</tr>
<tr>
<td>N 25</td>
<td>1.018 ± 0.5%</td>
<td>0.7399 ± 1.1%</td>
</tr>
<tr>
<td>N 30</td>
<td>1.016 ± 0.5%</td>
<td>1.043 ± 1.1%</td>
</tr>
<tr>
<td>N 40</td>
<td>1.011 ± 0.5%</td>
<td>1.450 ± 1.1%</td>
</tr>
</tbody>
</table>

Quotient folded o. same Spectrum
0.5 % > 10 keV
1 % < 10 keV

$\mu_{en}$ folded o. a spectrum
1% > 10 keV
2% < 10 keV
→ not totally independent
Measured Data (N20-N40)

N20

N25

N30

N40
## Results

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\Delta Q_{\text{Expo}}}{\Delta x}$</th>
<th>$K_w^{\text{Expo}}$</th>
<th>$K_{\text{air}}^{}$</th>
<th>$K_w^{\text{free-air}}$</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(10^{-12} , \text{C/mm})</td>
<td>(10^{-4} , \text{Gy})</td>
<td>(10^{-4} , \text{Gy})</td>
<td>(10^{-4} , \text{Gy})</td>
<td>±</td>
</tr>
<tr>
<td>N 20</td>
<td>2.075</td>
<td>3.263</td>
<td>8.083</td>
<td>3.319</td>
<td>1.017</td>
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<tr>
<td>N 25</td>
<td>3.869</td>
<td>6.067</td>
<td>8.275</td>
<td>6.123</td>
<td>1.009</td>
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<tr>
<td>N 30</td>
<td>3.649</td>
<td>5.709</td>
<td>5.446</td>
<td>5.680</td>
<td>0.995</td>
</tr>
<tr>
<td>N 40</td>
<td>4.821</td>
<td>7.508</td>
<td>5.135</td>
<td>7.446</td>
<td>0.991</td>
</tr>
<tr>
<td>rel. unc. [%]</td>
<td>rel. unc. [%]</td>
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<td>rel. unc. [%]</td>
<td>rel. unc. [%]</td>
<td>rel. unc. [%]</td>
</tr>
<tr>
<td>N 20</td>
<td>0.38</td>
<td>0.85</td>
<td>0.38</td>
<td>2.1</td>
<td>2.3</td>
</tr>
<tr>
<td>N 25</td>
<td>0.53</td>
<td>0.73</td>
<td>0.38</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td>N 30</td>
<td>0.56</td>
<td>0.76</td>
<td>0.38</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td>N 40</td>
<td>0.54</td>
<td>0.74</td>
<td>0.38</td>
<td>1.2</td>
<td>1.4</td>
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Summary and Conclusion

- Method to determine the water-kerma inside a graphite phantom
- Conversion factor (MC): $K_{w,g}(x) \rightarrow K_{w,g}(0)$
- Uncertainty $\sim 1\%$ (k=1)
- Phantom of water equivalent material $\rightarrow D_w$